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EXTENDING THE NUMBER OF COMPLETE SETS OF SSOFSs

By

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ABSTRACT

Complete sets of sum of squares orthogonal F-squares (SSOFSs) are presented for n = 8 = 2 × 4, n = 12 = 3 × 4, and n = 12 = 2 × 6. These were not obtained by Federer (2003) and hence represent an extension of the sum of squares geometry presented. Also, for all mutually orthogonal Latin square sets (MOLS(p^s, p^s - 1), p a prime number), complete sets of sum of squares orthogonal F-squares are available for n = $2^k \times p^s$. In addition, it is shown how to construct a set of sum of squares orthogonal Latin squares for all q (SSOLS(q, q - 1)) and illustrated with q = 6 and q = 10. The method of construction generalizes to any value of q. This is an extension of the theory in the area of projective geometry. These results represent a considerable extension of results available for constructing codes, fractional replicates, etc. with the sum of squares orthogonality property.

Key words: Regular F-square, semi-F-square, row frequency F-square, column frequency F-square, sum of squares orthogonality, combinatorial orthogonality, Latin square, factorial, main effect, interaction.

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1. INTRODUCTION

Federer (2003) was unable to obtain complete sets of sum of squares orthogonal F-squares (SSOFSs) for n = 8 = 2 × 4, n = 12 = 2 × 6, and n = 12 = 3 × 4. Complete sets of SSOFSs for these values of n are given herein. Further it is shown that the method used for n = 2 × 4 may be generalized to the case for n = $2^k \times p^s$, p a prime number. A new class of Latin squares that have sum of squares orthogonality but not combinatorial orthogonality is presented for all values of q. This class is denoted as SSOLS(q, q - 1).

Sum of squares orthogonality has been defined by Federer (2003) as follows. If the sum of the sums of squares and of the degrees of freedom for the F-squares formed from a factorial main effect or interaction add to that for the main effect or interaction, then the set is said to be sum of squares orthogonal and the set is called a sum of squares orthogonal F-squares (SSOFSs) set. These F-squares may be regular F-squares in that the symbols occur equally frequent in rows and in columns or they may be semi-F-squares. In a semi-F-square, the symbols occur equally frequent in a row or in a column but not both. Pesotan *et al.* (2003) have called these row frequency or column frequency squares, respectively.

In the next section, it is shown how to construct a complete set of sum of squares orthogonal F-squares for n = 8 = 2 × 4. The method of construction uses a set of mutually orthogonal Latin square of order p^s (MOLS(p^s, p^s - 1), p a prime number). The method generalizes to the case where $n = 2^k \times p^s$. In Section 3, it is shown how to construct a complete set of SSOFSs for $n = 12 = 3 \times 4$. A MOLS(4, 3) set is used in the construction. In Section 4, a complete set of SSOFSs is given for $n = 12 = 2 \times 6$. In Section 5, a new class of sum of squares orthogonal Latin squares is presented. Sum of squares orthogonality is used to define this class of Latin squares, SSOLS(q, q- 1). A SSOLS(6, 5) set is used to obtain the results in Section 4. The method for constructing the SSOLS(q, q - 1) set is described and illustrated for q = 6 (two forms of a Latin square) and q = 10.

2. COMPLETE SET OF SSOFSs FOR $n = 2 \times 4$

Let the levels of eight rows be the combination levels of the two-level (0, 1) factor A and the four-level (0, 1, 2, 3) factor B. Likewise, let the levels of the eight columns be the combination levels of the two-level (0, 1) factor C and the four-level (0, 1, 2, 3) factor D. The row × column interaction is made up the interactions of these four factors except for the two interactions A × B and C × D. The level of each of the four factors are given in Table 1 in the fourth, fifth, sixth, and seventh columns. The first column is the response variable y, the second column denotes the row number, and the third column denotes the column number in the 8 × 8 square.

F1 is formed by adding the levels of factors A and C, modulus 2.
F2 is formed by adding the levels of factors A and D, modulus 4.
F3 is formed by adding the levels of factors A, C, and D, modulus 4.
F4 is formed by adding the levels of factors B and C, modulus 4.
F11 is formed by adding the levels of factors A, B, and C, modulus 4.

The remaining F-squares contain the interaction of the factors B and D each at four levels, 0, 1, 2, and 3. The marks of the field for the number 4 are 0, 1, x, and x + 1. Certain addition and multiplication rules need to be followed for combinations of F-squares from the geometric components of the interactions BD, BD^x, and BD^{x+1}. The MOLS(4, 3) set was used for these geometric interactions. These were taken from Raghavarao (1971) and are:

Square 1				Square 2				Square 3			
0	1	x	x+1	0	x	x+1	1	0	x+1	1	x
1	0	x+1	x	1	x+1	x	0	1	x	0	x+1
x	x+1	0	1	x	0	1	x+1	x	1	x+1	0
x+1	x	1	0	x+1	1	0	x	x+1	0	x	1

Let x = 2 and x+1 = 3. Let the rows be the four-level B factor and let the columns be the four-level factor D. Then, the 16 combinations may be formed and be denoted by the number in the square. These numbers in the three squares correspond to the levels of the three geometric interactions BD, BD^x, and BD^{x+1}. The sum of the sums of squares and of the degrees of freedom for the three geometric interactions equals that for the B × D interaction with nine degrees of freedom. Each of the geometric interactions is associated with three degrees of freedom.

F5 is formed from the combinations in square 1 above.
F6 is formed from the combinations in square 2 above.

F7 is formed from the combinations in square 3 above.

From the interaction B × C × D with nine degrees of freedom, three F-squares are formed as follows:

F8 is formed from the numbers in square 1 plus the levels (0 or 1) of factor C, modulus 4.
F9 is formed from the numbers in square 2 plus the levels of factor C, modulus 4.
F10 is formed from the numbers in square 3 plus the levels of factor C, modulus 4.

From the interaction A × B × D with nine degrees of freedom, three F-squares are formed as follows:

F12 is formed from the numbers in square 1 plus the levels (0 or 1) of factor A, modulus 4.
F13 is formed from the numbers in square 2 plus the levels of factor A, modulus 4.
F14 is formed from the numbers in square 3 plus the levels of factor A, modulus 4.

From the interaction A × B × C × D with nine degrees of freedom, three F-squares are formed as follows:

F15 is formed from the numbers in square 1 plus the levels (0 or 1) of factors A and C, modulus 4.
F16 is formed from the numbers in square 2 plus the levels of factors A and C, modulus 4.
F17 is formed from the numbers in square 3 plus the levels of factors A and C, modulus 4.

As stated above, the B × D interaction is associated with the three F-squares F8, F9, and F10. These need to be entered in a data set in order to construct the remaining F-squares using a SAS program. This was not done here. The levels for all 17 F-squares were entered in the data set. The 17 F-squares given in the in the last 17 columns of the data set in Table 2.1.

Table 2.1. SAS code, levels of factors A, B, C, and D, and the 17 F-squares forming a complete set of SSOFSs.

```
DATA FSS8M;
INPUT y ROW COL A B C D F1 F2 F3 F4 F5 F6 F7 F8 F9 F10 F11F12 F13 F14 F15 F16 F17;
/* A*C=F1, A*D=F2, A*C*D=F3, B*C=F4, B*D=F5, F6, and F7, B*C*D=F8, F9, and F10,
A*B*C=F11, A*B*D= F12, F13, and F14, A*B*C*D=F15, F16, and F17*/

DATALINES;
92 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
66 1 2 0 0 0 1 0 1 1 0 1 2 3 1 2 3 0 1 2 3 1 2 3
19 1 3 0 0 0 2 0 2 2 0 2 3 1 2 3 1 0 2 3 1 2 3 1
19 1 4 0 0 0 3 0 3 3 0 3 1 2 3 1 2 0 3 1 2 3 1 2
29 1 5 0 0 1 0 1 0 1 1 0 0 0 1 1 1 1 0 0 0 1 1 1
16 1 6 0 0 1 1 1 1 2 1 1 2 3 2 3 0 1 1 2 3 2 3 0
25 1 7 0 0 1 2 1 2 3 1 2 3 1 3 0 2 1 2 3 1 3 0 2
25 1 8 0 0 1 3 1 3 0 1 3 1 2 0 2 3 1 3 1 2 0 2 3
60 2 1 0 1 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1
```

46	2	2	0	1	0	1	0	1	1	1	0	3	2	0	3	2	1	0	3	2	0	3	2
35	2	3	0	1	0	2	0	2	2	1	3	2	0	3	2	0	1	3	2	0	3	2	0
35	2	4	0	1	0	3	0	3	3	1	2	0	3	2	0	3	1	2	0	3	2	0	3
10	2	5	0	1	1	0	1	0	1	2	1	1	1	2	2	2	2	1	1	1	2	2	2
11	2	6	0	1	1	1	1	1	2	2	0	3	2	1	0	3	2	0	3	2	1	0	3
5	2	7	0	1	1	2	1	2	3	2	3	2	0	0	3	1	2	3	2	0	0	3	1
46	2	8	0	1	1	3	1	3	0	2	2	0	3	3	1	0	2	2	0	3	3	1	0
46	3	1	0	2	0	0	0	0	0	2	2	2	2	2	2	2	2	2	2	2	2	2	2
81	3	2	0	2	0	1	0	1	1	2	3	0	1	3	0	1	2	3	0	1	3	0	1
17	3	3	0	2	0	2	0	2	2	2	0	1	3	0	1	3	2	0	1	3	0	1	3
22	3	4	0	2	0	3	0	3	3	2	1	3	0	1	3	0	2	1	3	0	1	3	0
22	3	5	0	2	1	0	1	0	1	3	2	2	2	3	3	3	3	2	2	2	3	3	3
16	3	6	0	2	1	1	1	1	2	3	3	0	1	0	1	2	3	3	0	1	0	1	2
9	3	7	0	2	1	2	1	2	3	3	0	1	3	1	2	0	3	0	1	3	1	2	0
9	3	8	0	2	1	3	1	3	0	3	1	3	0	2	0	1	3	1	3	0	2	0	1
20	4	1	0	3	0	0	0	0	0	3	3	3	3	3	3	3	3	3	3	3	3	3	3
59	4	2	0	3	0	1	0	1	1	3	2	1	0	2	1	0	3	2	1	0	2	1	0
43	4	3	0	3	0	2	0	2	2	3	1	0	2	1	0	2	3	1	0	2	1	0	2
43	4	4	0	3	0	3	0	3	3	3	0	2	1	0	2	1	3	0	2	1	0	2	1
15	4	5	0	3	1	0	1	0	1	0	3	3	3	0	0	0	0	3	3	3	0	0	0
10	4	6	0	3	1	1	1	1	2	0	2	1	0	3	2	1	0	2	1	0	3	2	1
2	4	7	0	3	1	2	1	2	3	0	1	0	2	2	1	3	0	1	0	2	2	1	3
49	4	8	0	3	1	3	1	3	0	0	0	2	1	1	3	2	0	0	2	1	1	3	2
49	5	1	1	0	0	0	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1
64	5	2	1	0	0	1	1	2	2	0	1	2	3	1	2	3	1	2	3	0	2	3	0
25	5	3	1	0	0	2	1	3	3	0	2	3	1	2	3	1	1	3	0	2	3	0	2
25	5	4	1	0	0	3	1	0	0	0	3	1	2	3	1	2	1	0	2	3	0	2	3
24	5	5	1	0	1	0	0	1	2	1	0	0	0	1	1	1	2	1	1	1	2	2	2
8	5	6	1	0	1	1	0	2	3	1	1	2	3	2	3	0	2	2	3	0	3	0	1
7	5	7	1	0	1	2	0	3	0	1	2	3	1	3	0	2	2	3	0	2	0	1	3
7	5	8	1	0	1	3	0	0	1	1	3	1	2	0	2	3	2	0	2	3	1	3	0
34	6	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2
60	6	2	1	1	0	1	1	2	2	1	0	3	2	0	3	2	2	1	0	3	1	0	3
52	6	3	1	1	0	2	1	3	3	1	3	2	0	3	2	0	2	0	3	1	0	3	1
20	6	4	1	1	0	3	1	0	0	1	2	0	3	2	0	3	2	3	1	0	3	1	0
28	6	5	1	1	1	0	0	1	2	2	1	1	1	2	2	2	3	2	2	2	3	3	3
11	6	6	1	1	1	1	0	2	3	2	0	3	2	1	0	3	3	1	0	3	2	1	0
11	6	7	1	1	1	2	0	3	0	2	3	2	0	0	3	1	3	0	3	1	1	0	2
28	6	8	1	1	1	3	0	0	1	2	2	0	3	3	1	0	3	3	1	0	0	2	1
20	7	1	1	2	0	0	1	1	1	2	2	2	2	2	2	2	3	3	3	3	3	3	3
52	7	2	1	2	0	1	1	2	2	2	3	0	1	3	0	1	3	0	1	2	0	1	2
60	7	3	1	2	0	2	1	3	3	2	0	1	3	0	1	3	3	1	2	0	1	2	0
34	7	4	1	2	0	3	1	0	0	2	1	3	0	1	3	0	3	2	0	1	2	0	1
7	7	5	1	2	1	0	0	1	2	3	2	2	2	3	3	3	0	3	3	3	0	0	0
24	7	6	1	2	1	1	0	2	3	3	3	0	1	0	1	2	0	0	1	2	1	2	3
25	7	7	1	2	1	2	0	3	0	3	0	1	3	1	2	0	0	1	2	0	2	3	1

```
25 7 8 1 2 1 3 0 0 1 3 1 3 0 2 0 1 0 2 0 1 3 1 2
64 8 1 1 3 0 0 1 1 1 3 3 3 3 3 3 3 0 0 0 0 0 0 0
49 8 2 1 3 0 1 1 2 2 3 2 1 0 2 1 0 0 3 2 1 3 2 1
60 8 3 1 3 0 2 1 3 3 3 1 0 2 1 0 2 0 2 1 3 2 1 3
52 8 4 1 3 0 3 1 0 0 3 0 2 1 0 2 1 0 1 3 2 1 3 2
20 8 5 1 3 1 0 0 1 2 0 3 3 3 0 0 0 1 0 0 0 1 1 1
28 8 6 1 3 1 1 0 2 3 0 2 1 0 3 2 1 1 3 2 1 0 3 2
20 8 7 1 3 1 2 0 3 0 0 1 0 2 2 1 3 1 2 1 3 3 2 0
11 8 8 1 3 1 3 0 0 1 0 0 2 1 1 3 2 1 1 3 2 2 0 3
; RUN;
PROC PRINT;
RUN;
PROC GLM DATA=FSS8M;
 CLASS ROW COL A B C D;
 MODEL Y = A B A*B C D C*D A*C A*D A*C*D B*C B*D B*C*D A*B*C A*B*D
 A*B*C*D;
RUN;

PROC GLM DATA=FSS8M;
 CLASS ROW COL F1 F2 F3 F4 F5 F6 F7 F8 F9 F10 F11 F12 F13 F14
     F15 F16 F17;
 MODEL Y = ROW COL F1 F2 F3 F4 F5 F6 F7 F8 F9 F10 F11 F12 F13
       F14 F15 F16 F17;
RUN;
```

From the above SAS code, analysis of variance tables for the four factor factorial and for the 17 F-squares are obtained. The factorial analysis of variance is presented in Table 2.2. Note that the A and B effects and interaction are row effects and C and D and interaction are column effects. This is indicated in the table. The effects for these four factors and their interactions account for all the 64 degrees of freedom and sums of squares. The single degree of freedom sum of squares for the mean (correction for the mean) is 64(mean)^2 = 64(31.34375)^2 = 62,875.562. The total sum of squares of the 64 observations is 26,598.43750 + 62,875.562 = 89,474.

Table 2.2. Analysis of variance for the four factor factorial.

Source	DF	Sum of Squares	Mean Square
Model	63	26598.43750	422.19742
Error	0	0.00000	
Corrected Total	63	26598.43750	

Source	DF	Type I SS	Mean Square
ROW	7	898.43750	128.34821
A	1	0.06250	0.06250
B	3	190.06250	63.35417
A*B	3	708.31250	236.10417
COL	7	5273.43750	2181.91964

C	1	11025.00000	11025.00000
D	3	1344.81250	448.27083
C*D	3	2903.62500	967.87500
A*C	1	16.00000	16.00000
A*D	3	970.31250	323.43750
A*C*D	3	831.62500	277.20833
B*C	3	77.37500	25.79167
B*D	9	2705.56250	300.61806
B*C*D	9	1569.00000	174.33333
A*B*C	3	258.12500	86.04167
A*B*D	9	2193.31250	243.70139
A*B*C*D	9	1805.25000	200.58333

Since the factorial effects are orthogonal to each other the Type III sums of squares are the same as for Type I sums of squares. Note that the Type I sums of squares that are obtained by eliminating all effects above an effect in the table and ignoring all effects below it. This is called a nested analysis of variance. A Type III sum of squares is obtained eliminating any confounding with all of the other effects. The fact that the Type I and Type III sums of squares are identical indicates that the effects are both combinatorially and sums-of-squares orthogonal. An analysis of variance table for the sums of squares for the 17 F-squares is given in Table 2.3. Here the row and column effects are not partitioned. Note that the Type I and Type III sums of squares for F1 and F3 are identical. Also, F15, F16, and F17 sums of squares are identical as they should be since confounding with all other effects has been taken into account for both Type I and Type III calculations. In the Type I analysis, the total sum of squares for F-squares formed from an interaction is identical to the interaction sum of squares. For example, the sum of squares for F12 + F13 + F14 is 602.8125 + 1,011.6875 + 578.8125 = 2,193.3125 and that is the sum of squares for the A × B × D interaction. This holds for all sets of F-squares derived from an interaction. Also, all of the degrees of freedom and sums of squares are taken into account. Hence, this set of 17 F-squares is a complete and sum-of-squares orthogonal set of F-squares.

Table 2.3. Analysis of variance for the 17 F-squares

Source	DF	Sum of Squares	Mean Square		
Model	63	26598.43750	422.19742	.	.
Error	0	0.00000	.		
Corrected Total	63	26598.43750			

R-Square	Coeff Var	Root MSE	y Mean
1.000000	.	.	31.34375

Source	DF	Type I SS	Mean Square
ROW	7	898.43750	128.34821
COL	7	5273.43750	2181.91964
F1	1	16.00000	16.00000
F2	3	970.31250	323.43750
F3	3	831.62500	277.20833

F4	3	77.37500	25.79167
F5	3	718.31250	239.43750
F6	3	1467.18750	489.06250
F7	3	520.06250	173.35417
F8	3	1059.37500	353.12500
F9	3	331.25000	110.41667
F10	3	178.37500	59.45833
F11	3	258.12500	86.04167
F12	3	602.81250	200.93750
F13	3	1011.68750	337.22917
F14	3	578.81250	192.93750
F15	3	350.37500	116.79167
F16	3	853.25000	284.41667
F17	3	601.62500	200.54167

Source	DF	Type III SS	Mean Square
ROW	7	410.81250	58.68750
COL	7	15903.43750	2271.91964
F1	1	16.00000	16.00000
F2	3	313.62500	104.54167
F3	3	831.62500	277.20833
F4	3	152.37500	50.79167
F5	3	291.12500	97.04167
F6	3	1014.12500	338.04167
F7	3	272.75000	90.91667
F8	3	390.37500	130.12500
F9	3	786.50000	262.16667
F10	3	325.12500	108.37500
F11	3	258.12500	86.04167
F12	3	873.37500	291.12500
F13	3	1476.87500	492.29167
F14	3	914.75000	304.91667
F15	3	350.37500	116.79167
F16	3	853.25000	284.41667
F17	3	601.62500	200.54167

From the above procedure and results, the following theorem results:

Theorem: *For all sets of OLS(p^s, p^s - 1), there exists a complete set of sum-of-squares orthogonal F-squares for* $n = 2^k \times p^s$.

The proof is by construction. Note that sums of levels for 2^k modulus 2 are all 0 or 1. Hence, the above procedure follows directly. The p^s - 1 geometric interactions are formed as above for the two factors with p^s levels. The construction of F-squares follows that for $n = 8 = 2 \times 4$.

3. COMPLETE SET OF SSOFSs FOR n = 3 × 4

Let the row numbers 1 to 12 correspond to the 12 combinations of the factor A at three levels, 0, 1, 2, and the factor B at four levels, 0, 1, 2, 3. Likewise, let the 12 column numbers correspond to the 12 combinations of the factor C at three levels, 0, 1, 2, and the factor D at four levels, 0, 1, 2, 3. The row × column interaction may be partitioned into interaction effects among the four factors. These factorial interactions account for all the degrees of freedom and all of the sums of squares in the row × column interactions as shown in Table 3.2.

A SAS program for constructing all F-squares except those associated with the B × D interaction is given in Table 3.1. A SAS code was not found for constructing the three F-squares associated with this interaction. Hence, the symbols in these three were entered in the data set as F11, F12, and F13. "IF" and "THEN" statements are included in the SAS code to obtain the symbols for the various F-squares. Note that it was necessary to use a "-1" when 2*A or 2*C was used. The reason for this is unknown but it works to obtain a complete set of sum of squares orthogonal F-squares.

Table 3.1. SAS code for constructing a complete set of SSOFSs and analyses of variance tables.

```
data FSS3x4;
input Y ROW COL A B C D  F11 F12 F13 ;

F1 = A + C;  F2 = A + 2*C;  F1A=2*A+C; F2A=2*A+2*C;
F3 = A + D;  F4 = 2*A + D - 1;
F5 = F1+ D ; F6 = F2+ D;
F7 = F1A+ D;  F8=F2A+ D;
F9 = B + C;  F10 = B + 2*C - 1;
F14 =F11+C;  F15=F11+2*C-1;  F16=F12+C;
F17 =F12+2*C-1; F18=F13+C;  F19=F13+2*C-1;
F20 = B+F1;  F21 = B+F2;
F22 = B+F1A;  F23 = B+F2A;
F24 = F11 + A;  F25 = F11 + 2*A -1;
F26 = F12 + A;  F27 = F12 + 2*A - 1;
F28 = F13 + A;  F29 = F13 + 2*A - 1;
F30 = F24 + C;  F31 = F24 + 2*C -1;
F32 = F25 + C;  F33 = F25 + 2*C - 1;
F34 = F26 + C;  F35 = F26 + 2*C - 1;
F36 = F27 + C;  F37 = F27 + 2*C - 1;
F38 = F28 + C;  F39 = F28 + 2+C - 1;
F40 = F29 + C;  F41 = F29 + 2*C - 1;

IF F1>2 THEN F1=A+C-3; IF F2>2 THEN F2=A+2*C-3;
IF F1A>5 THEN F1A=2*A+C-6; IF F1A>2 THEN F1A = 2*A+C-3;
IF F2A>5 THEN F2A=2*A+2*C-6;  IF F2A>2 THEN F2A=2*A+2*C-3;
IF F3>3 THEN F3=A+D-4;
IF F4>3 THEN F4=2*A+D-1-4; IF F4=-1 THEN F4=2*A+D=3;
IF F5>3 THEN F5=F1+D-4; IF F5<0 THEN F5=F1+D;
```

```
IF F6>3 THEN F6=F2+D-4; IF F6<0 THEN F6=F2+D;
IF F7>3 THEN F7=F1A+D-4; IF F7<0 THEN F7=F1A+D;
IF F8>3 THEN F8=F2A+D-4; IF F8<0 THEN F8=F2A+D;
IF F9>3 THEN F9=B+C-4; IF F10 >3 THEN F10=B+2*C-1-4;
IF F10=-1 THEN F10=B+ 2*C=3;
IF F14>3 THEN F14=F11+C-4;
IF F15>3 THEN F15=F11+2*C-1-4;
IF F15=-1 THEN F15=F11+2*C=3;
IF F16>3 THEN F16=F12+C-4;
IF F17>3 THEN F17=F12+2*C-1-4; IF F17=-1 THEN F17=F12+2*C=3;
IF F18>3 THEN F18=F13+C-4;
IF F19>3 THEN F19=F13+2*C-1-4; IF F19=-1 THEN F19=F13+2*C=3;
IF F20>3 THEN F20=B+F1-4; F F20<0 THEN F20=B+F1;
IF F21>3 THEN F21=B+F2-4; IF F21<0 THEN F21=B+F2;
IF F22>3 THEN F22=B+F1A-4; IF F22<0 THEN F22=B+F1A;
IF F23>3 THEN F23=B+F2A-4; IF F23<0 THEN F23=B+F2A;
IF F24>3 THEN F24=F11+A-4;
IF F25>3 THEN F25=F11+2*A-1-4; IF F25=-1 THEN F25=F11+2*A=3;
IF F26>3 THEN F26=F12+A-4;
IF F27>3 THEN F27=F12+2*A-1-4; IF F27=-1 THEN F27=F12+2*A=3;
IF F28>3 THEN F28=F13+A-4;
IF F29>3 THEN F29=F13+2*A-1-4; IF F29=-1 THEN F29=F13+2*A=3;
IF F30>3 THEN F30=F11+F1-4; IF F30<0 THEN F30=F11+F1;
IF F31>3 THEN F31=F11+F2-4; IF F31<0 THEN F31=F11+F2;
IF F32>3 THEN F32=F11+F1A-4; IF F32<0 THEN F32=F11+F1A;
IF F33>3 THEN F33=F11+F2A-4; IF F33<0 THEN F33=F11+F2A;
IF F34>3 THEN F34=F12+F1-4; IF F34<0 THEN F34=F12+F1;
IF F35>3 THEN F35=F12+F2-4; IF F35<0 THEN F35=F12+F2;
IF F36>3 THEN F36=F12+F1A-4; IF F36<0 THEN F36=F12+F1A;
IF F37>3 THEN F37=F12+F2A-4; IF F37<0 THEN F37=F12+F2A;
IF F38>3 THEN F38=F13+F1-4; IF F38<0 THEN F38=F13+F1;
IF F39>3 THEN F39=F13+F2-4; IF F39<0 THEN F39=F13+F2;
IF F40>3 THEN F40=F13+F1A-4; IF F40<0 THEN F40=F13+F1A;
IF F41>3 THEN F41=F13+F2A-4; IF F41<0 THEN F41=F13+F2A;

/*F1 = A+ C, F2 = A + 2C, F3 = A+ D, F4 = 2A + 2D, F5 = F3 + C,
F6 = F3 + 2C, F7 = F4 + C, F8 = F4 + 2C, F9 = B + C,
F10 = B +2C, F11-13 FROM OLS(4, 3), F14-F19 = BCD,
F20-23 = ABC, F24-29 = ABD, F30-41 = ABCD*/

DATALINES;  /* 111213*/
10 1 1 0 0 0 0  0 0 0
11 1 2 0 0 0 1  1 2 3
12 1 3 0 0 0 2  2 3 1
13 1 4 0 0 0 3  3 1 2
14 1 5 0 0 1 0  0 0 0
```

15 1 6 0 0 1 1 1 2 3
16 1 7 0 0 1 2 2 3 1
17 1 8 0 0 1 3 3 1 2
18 1 9 0 0 2 0 0 0 0
18 1 10 0 0 2 1 1 2 3
18 1 11 0 0 2 2 2 3 1
18 1 12 0 0 2 3 3 1 2
19 2 1 0 1 0 0 1 1 1
20 2 2 0 1 0 1 0 3 2
25 2 3 0 1 0 2 3 2 0
99 2 4 0 1 0 3 2 0 3
55 2 5 0 1 1 0 1 1 1
55 2 6 0 1 1 1 0 3 2
55 2 7 0 1 1 2 3 2 0
45 2 8 0 1 1 3 2 0 3
46 2 9 0 1 2 0 1 1 1
47 2 10 0 1 2 1 0 3 2
47 2 11 0 1 2 2 3 2 0
47 2 12 0 1 2 3 2 0 3
47 3 1 0 2 0 0 2 2 2
31 3 2 0 2 0 1 3 0 1
33 3 3 0 2 0 2 0 1 3
56 3 4 0 2 0 3 1 3 0
56 3 5 0 2 1 0 2 2 2
56 3 6 0 2 1 1 3 0 1
78 3 7 0 2 1 2 0 1 3
81 3 8 0 2 1 3 1 3 0
34 3 9 0 2 2 0 2 2 2
45 3 10 0 2 2 1 3 0 1
45 3 11 0 2 2 2 0 1 3
54 3 12 0 2 2 3 1 3 0
54 4 1 0 3 0 0 3 3 3
54 4 2 0 3 0 1 2 1 0
12 4 3 0 3 0 2 1 0 2
15 4 4 0 3 0 3 0 2 1
45 4 5 0 3 1 0 3 3 3
46 4 6 0 3 1 1 2 1 0
56 4 7 0 3 1 2 1 0 2
65 4 8 0 3 1 3 0 2 1
66 4 9 0 3 2 0 3 3 3
46 4 10 0 3 2 1 2 1 0
47 4 11 0 3 2 2 1 0 2
48 4 12 0 3 2 3 0 2 1
10 5 1 1 0 0 0 0 0 0
11 5 2 1 0 0 1 1 2 3
12 5 3 1 0 0 2 2 3 1

13 5 4 1 0 0 3 3 1 2
14 5 5 1 0 1 0 0 0 0
15 5 6 1 0 1 1 1 2 3
16 5 7 1 0 1 2 2 3 1
17 5 8 1 0 1 3 3 1 2
18 5 9 1 0 2 0 0 0 0
18 5 10 1 0 2 1 1 2 3
18 5 11 1 0 2 2 2 3 1
18 5 12 1 0 2 3 3 1 2
19 6 1 1 1 0 0 1 1 1
20 6 2 1 1 0 1 0 3 2
25 6 3 1 1 0 2 3 2 0
99 6 4 1 1 0 3 2 0 3
55 6 5 1 1 1 0 1 1 1
55 6 6 1 1 1 1 0 3 2
55 6 7 1 1 1 2 3 2 0
45 6 8 1 1 1 3 2 0 3
46 6 9 1 1 2 0 1 1 1
47 6 10 1 1 2 1 0 3 2
47 6 11 1 1 2 2 3 2 0
47 6 12 1 1 2 3 2 0 3
47 7 1 1 2 0 0 2 2 2
31 7 2 1 2 0 1 3 0 1
33 7 3 1 2 0 2 0 1 3
56 7 4 1 2 0 3 1 3 0
56 7 5 1 2 1 0 2 2 2
56 7 6 1 2 1 1 3 0 1
78 7 7 1 2 1 2 0 1 3
81 7 8 1 2 1 3 1 3 0
34 7 9 1 2 2 0 2 2 2
45 7 10 1 2 2 1 3 0 1
45 7 11 1 2 2 2 0 1 3
54 7 12 1 2 2 3 1 3 0
54 8 1 1 3 0 0 3 3 3
54 8 2 1 3 0 1 2 1 0
12 8 3 1 3 0 2 1 0 2
15 8 4 1 3 0 3 0 2 1
45 8 5 1 3 1 0 3 3 3
46 8 6 1 3 1 1 2 1 0
56 8 7 1 3 1 2 1 0 2
65 8 8 1 3 1 3 0 2 1
66 8 9 1 3 2 0 3 3 3
46 8 10 1 3 2 1 2 1 0
47 8 11 1 3 2 2 1 0 2
48 8 12 1 3 2 3 0 2 1
47 9 1 2 0 0 0 0 0 0

31 9 2 2 0 0 1 1 2 3
33 9 3 2 0 0 2 2 3 1
56 9 4 2 0 0 3 3 1 2
56 9 5 2 0 1 0 0 0 0
56 9 6 2 0 1 1 1 2 3
78 9 7 2 0 1 2 2 3 1
81 9 8 2 0 1 3 3 1 2
34 9 9 2 0 2 0 0 0 0
45 9 10 2 0 2 1 1 2 3
45 9 11 2 0 2 2 2 3 1
54 9 12 2 0 2 3 3 1 2
54 10 1 2 1 0 0 1 1 1
54 10 2 2 1 0 1 0 3 2
12 10 3 2 1 0 2 3 2 0
15 10 4 2 1 0 3 2 0 3
45 10 5 2 1 1 0 1 1 1
46 10 6 2 1 1 1 0 3 2
56 10 7 2 1 1 2 3 2 0
65 10 8 2 1 1 3 2 0 3
66 10 9 2 1 2 0 1 1 1
46 10 10 2 1 2 1 0 3 2
47 10 11 2 1 2 2 3 2 0
48 10 12 2 1 2 3 2 0 3
47 11 1 2 2 0 0 2 2 2
31 11 2 2 2 0 1 3 0 1
33 11 3 2 2 0 2 0 1 3
56 11 4 2 2 0 3 1 3 0
56 11 5 2 2 1 0 2 2 2
56 11 6 2 2 1 1 3 0 1
78 11 7 2 2 1 2 0 1 3
81 11 8 2 2 1 3 1 3 0
34 11 9 2 2 2 0 2 2 2
45 11 10 2 2 2 1 3 0 1
45 11 11 2 2 2 2 0 1 3
54 11 12 2 2 2 3 1 3 0
54 12 1 2 3 0 0 3 3 3
54 12 2 2 3 0 1 2 1 0
12 12 3 2 3 0 2 1 0 2
15 12 4 2 3 0 3 0 2 1
45 12 5 2 3 1 0 3 3 3
46 12 6 2 3 1 1 2 1 0
56 12 7 2 3 1 2 1 0 2
65 12 8 2 3 1 3 0 2 1
66 12 9 2 3 2 0 3 3 3
46 12 10 2 3 2 1 2 1 0
47 12 11 2 3 2 2 1 0 2

```
48 12 12 2 3 2 3 0 2 1
; RUN;
PROC GLM DATA = FSS3x4;

CLASS ROW COL A B C D;

MODEL Y = ROW COL A*C A*D A*C*D B*C B*D B*C*D A*B*C A*B*D A*B*C*D;

RUN;

PROC GLM DATA = FSS3x4;

CLASS ROW COL A B C D F1 F2 F3 F4 F5 F6 F7 F8 F9 F10 F11 F12 F13
F14 F15 F16 F17 F18 F19 F20 F21 F22 F23 F24 F25 F26 F27 F28 F29
F30 F31 F32 F33 F34 F35 F36 F37 F38 F39 F40 F41;

MODEL Y = ROW COL F1 F2 F3 F4 F5 F6 F7 F8 F9 F10 F11 F12 F13
F14 F15 F16 F17 F18 F19 F20 F21 F22 F23 F24 F25 F26 F27 F28 F29
F30 F31 F32 F33 F34 F35 F36 F37 F38 F39 F40 F41;

RUN;
```

The output, somewhat edited, for the above SAS code is presented in Table 3.2. Thirty nine of the F-squares have four symbols (0, 1, 2, 3) and two, F1 and F2, have three symbols (0, 1, 2). These 41 F-squares appear in the last 41 columns of the first part of Table 3.2. Note that the Error sum of squares is zero as the total sum of squares is allocated to the mean, row, column, and F-square effects.

Table 3.2. Output of SAS code in Table 3.1.

obs	Y	ROW	COL	A	B	C	D	F11	F12	F13	F1	F2	F1A	F2A	F3	F4	F5	F6	F7	F8	F9	F10	F14	F15	F16	F17	F18	F19
1	10	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	11	1	2	0	0	0	1	1	2	3	0	0	0	0	1	0	1	1	1	1	0	0	1	0	2	1	3	2
3	12	1	3	0	0	0	2	2	3	1	0	0	0	0	2	1	2	2	2	2	0	0	2	1	3	2	1	0
4	13	1	4	0	0	0	3	3	1	2	0	0	0	0	3	2	3	3	3	3	0	0	3	2	1	0	2	1
5	14	1	5	0	0	1	0	0	0	0	1	2	1	2	0	0	1	2	1	2	1	1	1	1	1	1	1	1
6	15	1	6	0	0	1	1	1	2	3	1	2	1	2	1	0	2	3	2	3	1	1	2	2	3	3	0	0
7	16	1	7	0	0	1	2	2	3	1	1	2	1	2	2	1	3	0	3	0	1	1	3	3	0	0	2	2
8	17	1	8	0	0	1	3	3	1	2	1	2	1	2	3	2	0	1	0	1	1	1	0	0	2	2	3	3
9	18	1	9	0	0	2	0	0	0	0	2	1	2	1	0	0	2	1	2	1	2	3	2	3	2	3	2	3
10	18	1	10	0	0	2	1	1	2	3	2	1	2	1	1	0	3	2	3	2	2	3	3	0	0	1	1	2
11	18	1	11	0	0	2	2	2	3	1	2	1	2	1	2	1	0	3	0	3	2	3	0	1	1	2	3	0
12	18	1	12	0	0	2	3	3	1	2	2	1	2	1	3	2	1	0	1	0	2	3	1	2	3	0	0	1
13	19	2	1	0	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0
14	20	2	2	0	1	0	1	0	3	2	0	0	0	0	1	0	1	1	1	1	1	0	0	0	3	2	2	1
15	25	2	3	0	1	0	2	3	2	0	0	0	0	0	2	1	2	2	2	2	1	0	3	2	2	1	0	0
16	99	2	4	0	1	0	3	2	0	3	0	0	0	0	3	2	3	3	3	3	1	0	2	1	0	0	3	2
17	55	2	5	0	1	1	0	1	1	1	1	2	1	2	0	0	1	2	1	2	2	2	2	2	2	2	2	2
18	55	2	6	0	1	1	1	0	3	2	1	2	1	2	1	0	2	3	2	3	2	2	1	1	0	0	3	3
19	55	2	7	0	1	1	2	3	2	0	1	2	1	2	2	1	3	0	3	0	2	2	0	0	3	3	1	1
20	45	2	8	0	1	1	3	2	0	3	1	2	1	2	3	2	0	1	0	1	2	2	3	3	1	1	0	0
21	46	2	9	0	1	2	0	1	1	1	2	1	2	1	0	0	2	1	2	1	3	0	3	0	3	0	3	0
22	47	2	10	0	1	2	1	0	3	2	2	1	2	1	1	0	3	2	3	2	3	0	2	3	1	2	0	1
23	47	2	11	0	1	2	2	3	2	0	2	1	2	1	2	1	0	3	0	3	3	0	1	2	0	1	2	3
24	47	2	12	0	1	2	3	2	0	3	2	1	2	1	3	2	1	0	1	0	3	0	0	1	2	3	1	2

Obs	F20	F21	F22	F23	F24	F25	F26	F27	F28	F29	F30	F31	F32	F33	F34	F35	F36	F37	F38	F39	F40	F41
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
2	0	0	0	0	1	0	2	1	3	2	1	0	0	1	2	1	1	0	3	3	2	1
3	0	0	0	0	2	1	3	2	1	0	2	1	1	0	3	2	2	1	1	2	0	1
4	0	0	0	0	3	2	1	0	2	1	3	2	2	1	1	0	0	1	2	3	1	0
5	1	2	1	2	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1	2	0	0
6	1	2	1	2	1	0	2	1	3	2	2	2	1	1	3	3	2	2	0	1	3	3
7	1	2	1	2	2	1	3	2	1	0	3	3	2	2	0	1	3	3	2	3	1	1
8	1	2	1	2	3	2	1	0	2	1	0	1	3	3	2	2	1	1	3	0	2	2
9	2	1	2	1	0	0	0	0	0	0	2	3	1	2	2	3	1	2	2	3	1	2
10	2	1	2	1	1	0	2	1	3	2	3	2	2	3	0	3	3	3	1	0	1	0
11	2	1	2	1	2	1	3	2	1	0	0	3	3	3	1	0	1	0	3	2	2	3
12	2	1	2	1	3	2	1	0	2	1	1	0	1	0	3	2	2	3	0	3	3	3
13	1	1	1	1	1	0	1	0	1	0	1	0	0	1	1	0	0	1	1	2	0	1
14	1	1	1	1	0	0	3	2	2	1	0	0	0	0	3	2	2	1	2	3	1	0
15	1	1	1	1	3	2	2	1	0	0	3	2	2	1	2	1	1	0	0	1	0	0
16	1	1	1	1	2	1	0	0	3	2	2	1	1	0	0	0	0	0	3	3	2	1
17	2	3	2	3	1	0	1	0	1	0	2	2	1	1	2	2	1	1	2	3	1	1
18	2	3	2	3	0	0	3	2	2	1	1	1	0	0	0	1	3	3	3	0	2	2
19	2	3	2	3	3	2	2	1	0	0	0	1	3	3	3	3	2	2	1	2	0	0
20	2	3	2	3	2	1	0	0	3	2	3	3	2	2	1	1	0	0	0	1	3	3
21	3	2	3	2	1	0	1	0	1	0	3	2	2	3	3	2	2	3	3	2	2	3
22	3	2	3	2	0	0	3	2	2	1	2	3	1	2	1	0	1	0	0	3	3	3
23	3	2	3	2	3	2	2	1	0	0	1	0	1	0	0	3	3	3	2	3	1	2
24	3	2	3	2	2	1	0	0	3	2	0	3	3	3	2	3	1	2	1	0	1	0

Obs	Y	ROW	COL	A	B	C	D	F11	F12	F13	F1	F2	F1A	F2A	F3	F4	F5	F6	F7	F8	F9	F10	F14	F15	F16	F17	F18	F19
25	47	3	1	0	2	0	0	2	2	2	0	0	0	0	0	0	0	0	0	0	2	1	2	1	2	1	2	1
26	31	3	2	0	2	0	1	3	0	1	0	0	0	0	1	0	1	1	1	1	2	1	3	2	0	0	1	0
27	33	3	3	0	2	0	2	0	1	3	0	0	0	0	2	1	2	2	2	2	2	1	0	0	1	0	3	2
28	56	3	4	0	2	0	3	1	3	0	0	0	0	0	3	2	3	3	3	3	2	1	1	0	3	2	0	0
29	56	3	5	0	2	1	0	2	2	2	1	2	1	2	0	0	1	2	1	2	3	3	3	3	3	3	3	3
30	56	3	6	0	2	1	1	3	0	1	1	2	1	2	1	0	2	3	2	3	3	3	0	0	1	1	2	2
31	78	3	7	0	2	1	2	0	1	3	1	2	1	2	2	1	3	0	3	0	3	3	1	1	2	2	0	0
32	81	3	8	0	2	1	3	1	3	0	1	2	1	2	3	2	0	1	0	1	3	3	2	2	0	0	1	1
33	34	3	9	0	2	2	0	2	2	2	2	1	2	1	0	0	2	1	2	1	0	1	0	1	0	1	0	1
34	45	3	10	0	2	2	1	3	0	1	2	1	2	1	1	0	3	2	3	2	0	1	1	2	2	3	3	0
35	45	3	11	0	2	2	2	0	1	3	2	1	2	1	2	1	0	3	0	3	0	1	2	3	3	0	1	2
36	54	3	12	0	2	2	3	1	3	0	2	1	2	1	3	2	1	0	1	0	0	1	3	0	1	2	2	3
37	54	4	1	0	3	0	0	3	3	3	0	0	0	0	0	0	0	0	0	0	3	2	3	2	3	2	3	2
38	54	4	2	0	3	0	1	2	1	0	0	0	0	0	1	0	1	1	1	1	3	2	2	1	1	0	0	0
39	12	4	3	0	3	0	2	1	0	2	0	0	0	0	2	1	2	2	2	2	3	2	1	0	0	0	2	1
40	15	4	4	0	3	0	3	0	2	1	0	0	0	0	3	2	3	3	3	3	3	2	0	0	2	1	1	0
41	45	4	5	0	3	1	0	3	3	3	1	2	1	2	0	0	1	2	1	2	0	0	0	0	0	0	0	0
42	46	4	6	0	3	1	1	2	1	0	1	2	1	2	1	0	2	3	2	3	0	0	3	3	2	2	1	1
43	56	4	7	0	3	1	2	1	0	2	1	2	1	2	2	1	3	0	3	0	0	0	2	2	1	1	3	3
44	65	4	8	0	3	1	3	0	2	1	1	2	1	2	3	2	0	1	0	1	0	0	1	1	3	3	2	2
45	66	4	9	0	3	2	0	3	3	3	2	1	2	1	0	0	2	1	2	1	1	2	1	2	1	2	1	2
46	46	4	10	0	3	2	1	2	1	0	2	1	2	1	1	0	3	2	3	2	1	2	0	1	3	0	2	3
47	47	4	11	0	3	2	2	1	0	2	2	1	2	1	2	1	0	3	0	3	1	2	3	0	2	3	0	1
48	48	4	12	0	3	2	3	0	2	1	2	1	2	1	3	2	1	0	1	0	1	2	2	3	0	1	3	0

Obs	F20	F21	F22	F23	F24	F25	F26	F27	F28	F29	F30	F31	F32	F33	F34	F35	F36	F37	F38	F39	F40	F41
25	2	2	2	2	2	1	2	1	2	1	2	1	1	0	2	1	1	0	2	3	1	0
26	2	2	2	2	3	2	0	0	1	0	3	2	2	1	0	0	0	0	1	2	0	1
27	2	2	2	2	0	0	1	0	3	2	0	0	0	0	1	0	0	1	3	3	2	1
28	2	2	2	2	1	0	3	2	0	0	1	0	0	1	3	2	2	1	0	1	0	0
29	3	0	3	0	2	1	2	1	2	1	3	3	2	2	3	3	2	2	3	0	2	2
30	3	0	3	0	3	2	0	0	1	0	0	1	3	3	1	1	0	0	2	3	1	1
31	3	0	3	0	0	0	1	0	3	2	1	1	0	0	2	2	1	1	0	1	3	3
32	3	0	3	0	1	0	3	2	0	0	2	2	1	1	0	1	3	3	1	2	0	0
33	0	3	0	3	2	1	2	1	2	1	0	3	3	3	0	3	3	3	0	3	3	3
34	0	3	0	3	3	2	0	0	1	0	1	0	1	0	2	3	1	2	3	2	2	3
35	0	3	0	3	0	0	1	0	3	2	2	3	1	2	3	2	2	3	1	0	1	0
36	0	3	0	3	1	0	3	2	0	0	3	2	2	3	1	0	1	0	2	3	1	2
37	3	3	3	3	3	2	3	2	3	2	3	2	2	1	3	2	2	1	3	3	2	1
38	3	3	3	3	2	1	1	0	0	0	2	1	1	0	1	0	0	1	0	1	0	0
39	3	3	3	3	1	0	0	0	2	1	1	0	0	1	0	0	0	0	2	3	1	0
40	3	3	3	3	0	0	2	1	1	0	0	0	0	0	2	1	1	0	1	2	0	1
41	0	1	0	1	3	2	3	2	3	2	0	1	3	3	0	1	3	3	0	1	3	3
42	0	1	0	1	2	1	1	0	0	0	3	3	2	2	2	2	1	1	1	2	0	0
43	0	1	0	1	1	0	0	0	2	1	2	2	1	1	1	1	0	0	3	0	2	2
44	0	1	0	1	0	0	2	1	1	0	1	1	0	0	3	3	2	2	2	3	1	1
45	1	0	1	0	3	2	3	2	3	2	1	0	1	0	1	0	1	0	1	0	1	0
46	1	0	1	0	2	1	1	0	0	0	0	3	3	3	3	2	2	3	2	3	1	2
47	1	0	1	0	1	0	0	0	2	1	3	2	2	3	2	3	1	2	0	3	3	3
48	1	0	1	0	0	0	2	1	1	0	2	3	1	2	0	3	3	3	3	2	2	3

Obs	Y	ROW	COL	A	B	C	D	F11	F12	F13	F1	F2	F1A	F2A	F3	F4	F5	F6	F7	F8	F9	F10	F14	F15	F16	F17	F18	F19
49	10	5	1	1	0	0	0	0	0	0	1	1	2	2	1	1	1	1	2	2	0	0	0	0	0	0	0	0
50	11	5	2	1	0	0	1	1	2	3	1	1	2	2	2	2	2	2	3	3	0	0	1	0	2	1	3	2
51	12	5	3	1	0	0	2	2	3	1	1	1	2	2	3	3	3	3	0	0	0	0	2	1	3	2	1	0
52	13	5	4	1	0	0	3	3	1	2	1	1	2	2	0	0	0	0	1	1	0	0	3	2	1	0	2	1
53	14	5	5	1	0	1	0	0	0	0	2	0	0	1	1	1	2	3	3	1	1	1	1	1	1	1	1	1
54	15	5	6	1	0	1	1	1	2	3	2	0	0	1	2	2	3	1	1	2	1	1	2	2	3	3	0	0
55	16	5	7	1	0	1	2	2	3	1	2	0	0	1	3	3	0	2	2	3	1	1	3	3	0	0	2	2
56	17	5	8	1	0	1	3	3	1	2	2	0	0	1	0	0	1	3	3	0	1	1	0	0	2	2	3	3
57	18	5	9	1	0	2	0	0	0	0	0	2	1	0	1	1	3	2	1	0	2	3	2	3	2	3	2	3
58	18	5	10	1	0	2	1	1	2	3	0	2	1	0	2	2	1	3	2	1	2	3	3	0	0	1	1	2
59	18	5	11	1	0	2	2	2	3	1	0	2	1	0	3	3	2	0	3	2	2	3	0	1	1	2	3	0
60	18	5	12	1	0	2	3	3	1	2	0	2	1	0	0	0	3	1	0	3	2	3	1	2	3	0	0	1
61	19	6	1	1	1	0	0	1	1	1	1	1	2	2	1	1	1	1	2	2	1	0	1	0	1	0	1	0
62	20	6	2	1	1	0	1	0	3	2	1	1	2	2	2	2	2	2	3	3	1	0	0	0	3	2	2	1
63	25	6	3	1	1	0	2	3	2	0	1	1	2	2	3	3	3	3	0	0	1	0	3	2	2	1	0	0
64	99	6	4	1	1	0	3	2	0	3	1	1	2	2	0	0	0	0	1	1	1	0	2	1	0	0	3	2
65	55	6	5	1	1	1	0	1	1	1	2	0	0	1	1	1	2	3	3	1	2	2	2	2	2	2	2	2
66	55	6	6	1	1	1	1	0	3	2	2	0	0	1	2	2	3	1	1	2	2	2	1	1	0	0	3	3
67	55	6	7	1	1	1	2	3	2	0	2	0	0	1	3	3	0	2	2	3	2	2	0	0	3	3	1	1
68	45	6	8	1	1	1	3	2	0	3	2	0	0	1	0	0	1	3	3	0	2	2	3	3	1	1	0	0
69	46	6	9	1	1	2	0	1	1	1	0	2	1	0	1	1	3	2	1	0	3	0	3	0	3	0	3	0
70	47	6	10	1	1	2	1	0	3	2	0	2	1	0	2	2	1	3	2	1	3	0	2	3	1	2	0	1
71	47	6	11	1	1	2	2	3	2	0	0	2	1	0	3	3	2	0	3	2	3	0	1	2	0	1	2	3
72	47	6	12	1	1	2	3	2	0	3	0	2	1	0	0	0	3	1	0	3	3	0	0	1	2	3	1	2

Obs	F20	F21	F22	F23	F24	F25	F26	F27	F28	F29	F30	F31	F32	F33	F34	F35	F36	F37	F38	F39	F40	F41
49	1	1	2	2	1	1	1	1	1	1	1	0	1	0	1	0	1	0	1	2	1	0
50	1	1	2	2	2	2	3	3	0	0	2	1	2	1	3	2	3	2	0	0	1	3
51	1	1	2	2	3	3	0	0	2	2	3	2	3	2	0	3	1	3	2	3	2	1
52	1	1	2	2	0	0	2	2	3	3	0	3	1	3	2	1	2	1	3	3	3	2
53	2	3	3	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	3	2	2
54	2	3	3	1	2	2	3	3	0	0	3	3	3	3	0	2	2	3	1	3	3	0
55	2	3	3	1	3	3	0	0	2	2	0	2	2	3	1	3	3	0	3	1	3	3
56	2	3	3	1	0	0	2	2	3	3	1	3	3	0	3	3	3	3	0	2	2	3
57	3	2	1	0	1	1	1	1	1	1	3	2	3	0	3	2	3	0	3	2	3	0
58	3	2	1	0	2	2	3	3	0	0	1	3	2	1	2	0	3	2	3	1	0	3
59	3	2	1	0	3	3	0	0	2	2	2	0	3	2	3	1	0	3	1	3	2	1
60	3	2	1	0	0	0	2	2	3	3	3	1	0	3	1	3	2	1	2	0	3	2
61	2	2	3	3	2	2	2	2	2	2	2	1	2	1	2	1	2	1	2	3	2	1
62	2	2	3	3	1	1	0	0	3	3	1	0	1	0	0	3	1	3	3	3	3	2
63	2	2	3	3	0	0	3	3	1	1	0	3	1	3	3	2	3	2	1	2	1	0
64	2	2	3	3	3	3	1	1	0	0	3	2	3	2	1	0	1	0	0	0	1	3
65	3	1	1	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	1	3	3
66	3	1	1	2	1	1	0	0	3	3	2	2	2	2	1	3	3	0	0	2	2	3
67	3	1	1	2	0	0	3	3	1	1	1	3	3	0	0	2	2	3	2	3	2	2
68	3	1	1	2	3	3	1	1	0	0	0	2	2	3	2	2	2	2	1	3	3	0
69	1	3	2	1	2	2	2	2	2	2	1	3	2	1	1	3	2	1	1	3	2	1
70	1	3	2	1	1	1	0	0	3	3	3	2	3	0	3	1	0	3	2	0	3	2
71	1	3	2	1	0	0	3	3	1	1	3	1	0	3	2	0	3	2	3	2	3	0
72	1	3	2	1	3	3	1	1	0	0	2	0	3	2	3	2	3	0	3	1	0	3

Obs	Y	ROW	COL	A	B	C	D	F11	F12	F13	F1	F2	F1A	F2A	F3	F4	F5	F6	F7	F8	F9	F10	F14	F15	F16	F17	F18	F19
73	47	7	1	1	2	0	0	2	2	2	1	1	2	2	1	1	1	1	2	2	2	1	2	1	2	1	2	1
74	31	7	2	1	2	0	1	3	0	1	1	1	2	2	2	2	2	2	3	3	2	1	3	2	0	0	1	0
75	33	7	3	1	2	0	2	0	1	3	1	1	2	2	3	3	3	3	0	0	2	1	0	0	1	0	3	2
76	56	7	4	1	2	0	3	1	3	0	1	1	2	2	0	0	0	0	1	1	2	1	1	0	3	2	0	0
77	56	7	5	1	2	1	0	2	2	2	2	0	0	1	1	1	2	3	3	1	3	3	3	3	3	3	3	3
78	56	7	6	1	2	1	1	3	0	1	2	0	0	1	2	2	3	1	1	2	3	3	0	0	1	1	2	2
79	78	7	7	1	2	1	2	0	1	3	2	0	0	1	3	3	0	2	2	3	3	3	1	1	2	2	0	0
80	81	7	8	1	2	1	3	1	3	0	2	0	0	1	0	0	1	3	3	0	3	3	2	2	0	0	1	1
81	34	7	9	1	2	2	0	2	2	2	0	2	1	0	1	1	3	2	1	0	0	1	0	1	0	1	0	1
82	45	7	10	1	2	2	1	3	0	1	0	2	1	0	2	2	1	3	2	1	0	1	1	2	2	3	3	0
83	45	7	11	1	2	2	2	0	1	3	0	2	1	0	3	3	2	0	3	2	0	1	2	3	3	0	1	2
84	54	7	12	1	2	2	3	1	3	0	0	2	1	0	0	0	3	1	0	3	0	1	3	0	1	2	2	3
85	54	8	1	1	3	0	0	3	3	3	1	1	2	2	1	1	1	1	2	2	3	2	3	2	3	2	3	2
86	54	8	2	1	3	0	1	2	1	0	1	1	2	2	2	2	2	2	3	3	3	2	2	1	1	0	0	0
87	12	8	3	1	3	0	2	1	0	2	1	1	2	2	3	3	3	3	0	0	3	2	1	0	0	0	2	1
88	15	8	4	1	3	0	3	0	2	1	1	1	2	2	0	0	0	0	1	1	3	2	0	0	2	1	1	0
89	45	8	5	1	3	1	0	3	3	3	2	0	0	1	1	1	2	3	3	1	0	0	0	0	0	0	0	0
90	46	8	6	1	3	1	1	2	1	0	2	0	0	1	2	2	3	1	1	2	0	0	3	3	2	2	1	1
91	56	8	7	1	3	1	2	1	0	2	2	0	0	1	3	3	0	2	2	3	0	0	2	2	1	1	3	3
92	65	8	8	1	3	1	3	0	2	1	2	0	0	1	0	0	1	3	3	0	0	0	1	1	3	3	2	2
93	66	8	9	1	3	2	0	3	3	3	0	2	1	0	1	1	3	2	1	0	1	2	1	2	1	2	1	2
94	46	8	10	1	3	2	1	2	1	0	0	2	1	0	2	2	1	3	2	1	1	2	0	1	3	0	2	3

Obs	Y	ROW	COL	A	B	C	D	F11	F12	F13	F1	F2	F1A	F2A	F3	F4	F5	F6	F7	F8	F9	F10	F14	F15	F16	F17	F18	F19
95	47	8	11	1	3	2	2	1	0	2	0	2	1	0	3	3	2	0	3	2	1	2	3	0	2	3	0	1
96	48	8	12	1	3	2	3	0	2	1	0	2	1	0	0	0	3	1	0	3	1	2	2	3	0	1	3	0

Obs	F20	F21	F22	F23	F24	F25	F26	F27	F28	F29	F30	F31	F32	F33	F34	F35	F36	F37	F38	F39	F40	F41
73	3	3	0	0	3	3	3	3	3	3	3	2	3	2	3	2	3	2	3	3	3	2
74	3	3	0	0	0	0	1	1	2	2	0	3	1	3	1	0	1	0	2	3	2	1
75	3	3	0	0	1	1	2	2	0	0	1	0	1	0	2	1	2	1	0	0	1	3
76	3	3	0	0	2	2	0	0	1	1	2	1	2	1	0	3	1	3	1	2	1	0
77	0	2	2	3	3	3	3	3	3	3	0	2	2	3	0	2	2	3	0	2	2	3
78	0	2	2	3	0	0	1	1	2	2	1	3	3	0	2	2	2	2	3	1	3	3
79	0	2	2	3	1	1	2	2	0	0	2	2	2	2	3	3	3	3	1	3	3	0
80	0	2	2	3	2	2	0	0	1	1	3	3	3	3	1	3	3	0	2	3	2	2
81	2	0	3	2	3	3	3	3	3	3	2	0	3	2	2	0	3	2	2	0	3	2
82	2	0	3	2	0	0	1	1	2	2	3	1	0	3	3	2	3	0	1	3	2	1
83	2	0	3	2	1	1	2	2	0	0	3	2	3	0	1	3	2	1	3	1	0	3
84	2	0	3	2	2	2	0	0	1	1	1	3	2	1	3	1	0	3	3	2	3	0
85	0	0	1	1	0	0	0	0	0	0	0	3	1	3	0	3	1	3	0	0	1	3
86	0	0	1	1	3	3	2	2	1	1	3	2	3	2	2	1	2	1	1	2	1	0
87	0	0	1	1	2	2	1	1	3	3	2	1	2	1	1	0	1	0	3	3	3	2
88	0	0	1	1	1	1	3	3	2	2	1	0	1	0	3	2	3	2	2	3	2	1
89	1	3	3	0	0	0	0	0	0	0	1	3	3	0	1	3	3	0	1	3	3	0
90	1	3	3	0	3	3	2	2	1	1	0	2	2	3	3	3	3	3	2	3	2	2
91	1	3	3	0	2	2	1	1	3	3	3	3	3	3	2	2	2	2	0	2	2	3
92	1	3	3	0	1	1	3	3	2	2	2	2	2	2	0	2	2	3	3	1	3	3
93	3	1	0	3	0	0	0	0	0	0	3	1	0	3	3	1	0	3	3	1	0	3
94	3	1	0	3	3	3	2	2	1	1	2	0	3	2	1	3	2	1	3	2	3	0
95	3	1	0	3	2	2	1	1	3	3	1	3	2	1	3	2	3	0	2	0	3	2
96	3	1	0	3	1	1	3	3	2	2	3	2	3	0	2	0	3	2	1	3	2	1

Obs	Y	ROW	COL	A	B	C	D	F11	F12	F13	F1	F2	F1A	F2A	F3	F4	F5	F6	F7	F8	F9	F10	F14	F15	F16	F17	F18	F19
97	47	9	1	2	0	0	0	0	0	0	2	2	1	1	2	3	2	2	1	1	0	0	0	0	0	0	0	0
98	31	9	2	2	0	0	1	1	2	3	2	2	1	1	3	0	3	3	2	2	0	0	1	0	2	1	3	2
99	33	9	3	2	0	0	2	2	3	1	2	2	1	1	0	1	0	0	3	3	0	0	2	1	3	2	1	0
100	56	9	4	2	0	0	3	3	1	2	2	2	1	1	1	2	1	1	0	0	0	0	3	2	1	0	2	1
101	56	9	5	2	0	1	0	0	0	0	0	1	2	0	2	3	3	1	2	0	1	1	1	1	1	1	1	1
102	56	9	6	2	0	1	1	1	2	3	0	1	2	0	3	0	1	2	3	1	1	1	2	2	3	3	0	0
103	78	9	7	2	0	1	2	2	3	1	0	1	2	0	0	1	2	3	0	2	1	1	3	3	0	0	2	2
104	81	9	8	2	0	1	3	3	1	2	0	1	2	0	1	2	3	0	1	3	1	1	0	0	2	2	3	3
105	34	9	9	2	0	2	0	0	0	0	1	3	0	2	2	3	1	3	0	2	2	3	2	3	2	3	2	3
106	45	9	10	2	0	2	1	1	2	3	1	3	0	2	3	0	2	0	1	3	2	3	3	0	0	1	1	2
107	45	9	11	2	0	2	2	2	3	1	1	3	0	2	0	1	3	1	2	0	2	3	0	1	1	2	3	0
108	54	9	12	2	0	2	3	3	1	2	1	3	0	2	1	2	0	2	3	1	2	3	1	2	3	0	0	1
109	54	10	1	2	1	0	0	1	1	1	2	2	1	1	2	3	2	2	1	1	1	0	1	0	1	0	1	0
110	54	10	2	2	1	0	1	0	3	2	2	2	1	1	3	0	3	3	2	2	1	0	0	0	3	2	2	1
111	12	10	3	2	1	0	2	3	2	0	2	2	1	1	0	1	0	0	3	3	1	0	3	2	2	1	0	0
112	15	10	4	2	1	0	3	2	0	3	2	2	1	1	1	2	1	1	0	0	1	0	2	1	0	0	3	2
113	45	10	5	2	1	1	0	1	1	1	0	1	2	0	2	3	3	1	2	0	2	2	2	2	2	2	2	2
114	46	10	6	2	1	1	1	0	3	2	0	1	2	0	3	0	1	2	3	1	2	2	1	1	0	0	3	3
115	56	10	7	2	1	1	2	3	2	0	0	1	2	0	0	1	2	3	0	2	2	2	0	0	3	3	1	1
116	65	10	8	2	1	1	3	2	0	3	0	1	2	0	1	2	3	0	1	3	2	2	3	3	1	1	0	0
117	66	10	9	2	1	2	0	1	1	1	1	3	0	2	2	3	1	3	0	2	3	0	3	0	3	0	3	0
118	46	10	10	2	1	2	1	0	3	2	1	3	0	2	3	0	2	0	1	3	3	0	2	3	1	2	0	1
119	47	10	11	2	1	2	2	3	2	0	1	3	0	2	0	1	3	1	2	0	3	0	1	2	0	1	2	3
120	48	10	12	2	1	2	3	2	0	3	1	3	0	2	1	2	0	2	3	1	3	0	0	1	2	3	1	2

Obs	F20	F21	F22	F23	F24	F25	F26	F27	F28	F29	F30	F31	F32	F33	F34	F35	F36	F37	F38	F39	F40	F41
97	2	2	1	1	2	3	2	3	2	3	2	1	3	2	2	1	3	2	2	3	3	2
98	2	2	1	1	3	0	0	1	1	2	3	2	2	3	0	3	3	3	1	1	0	0
99	2	2	1	1	0	1	1	2	3	0	0	3	3	3	1	1	0	0	3	3	2	3
100	2	2	1	1	1	2	3	0	0	1	1	1	0	0	3	2	2	3	0	0	3	3
101	3	1	2	0	2	3	2	3	2	3	3	3	2	0	3	3	2	0	3	1	2	0
102	3	1	2	0	3	0	0	1	1	2	1	2	3	1	2	3	0	2	3	0	1	3
103	3	1	2	0	0	1	1	2	3	0	2	3	0	2	3	0	1	3	1	2	3	1
104	3	1	2	0	1	2	3	0	0	1	3	0	1	3	1	2	3	1	2	3	0	2
105	1	3	0	2	2	3	2	3	2	3	1	3	0	2	1	3	0	2	1	3	0	2
106	1	3	0	2	3	0	0	1	1	2	2	0	1	3	3	1	2	0	0	2	3	1
107	1	3	0	2	0	1	1	2	3	0	3	1	2	0	0	2	3	1	2	0	1	3
108	1	3	0	2	1	2	3	0	0	1	0	2	3	1	2	0	1	3	3	1	2	0
109	3	3	2	2	3	0	3	0	3	0	3	2	2	3	3	2	2	3	3	3	2	3
110	3	3	2	2	2	3	1	2	0	1	2	1	3	2	1	1	0	0	0	0	3	3
111	3	3	2	2	1	2	0	1	2	3	1	1	0	0	0	3	3	3	2	3	3	2
112	3	3	2	2	0	1	2	3	1	2	0	3	3	3	2	1	3	2	1	1	0	0
113	1	2	3	1	3	0	3	0	3	0	1	2	3	1	1	2	3	1	1	2	3	1
114	1	2	3	1	2	3	1	2	0	1	3	3	2	0	3	0	1	3	2	3	0	2
115	1	2	3	1	1	2	0	1	2	3	3	0	1	3	2	3	0	2	3	1	2	0
116	1	2	3	1	0	1	2	3	1	2	2	3	0	2	3	3	2	0	3	0	1	3
117	2	0	1	3	3	0	3	0	3	0	2	0	1	3	2	0	1	3	2	0	1	3
118	2	0	1	3	2	3	1	2	0	1	1	3	0	2	0	2	3	1	3	1	2	0

119	2	0	1	3	1	2	0	1	2	3	0	2	3	1	3	1	2	0	1	3	0	2
120	2	0	1	3	0	1	2	3	1	2	3	1	2	0	1	3	0	2	0	2	3	1

Obs	Y	ROW	COL	A	B	C	D	F11	F12	F13	F1	F2	F1A	F2A	F3	F4	F5	F6	F7	F8	F9	F10	F14	F15	F16	F17	F18	F19
121	47	11	1	2	2	0	0	2	2	2	2	2	1	1	2	3	2	2	1	1	2	1	2	1	2	1	2	1
122	31	11	2	2	2	0	1	3	0	1	2	2	1	1	3	0	3	3	2	2	2	1	3	2	0	0	1	0
123	33	11	3	2	2	0	2	0	1	3	2	2	1	1	0	1	0	0	3	3	2	1	0	0	1	0	3	2
124	56	11	4	2	2	0	3	1	3	0	2	2	1	1	1	2	1	1	0	0	2	1	1	0	3	2	0	0
125	56	11	5	2	2	1	0	2	2	2	0	1	2	0	2	3	3	1	2	0	3	3	3	3	3	3	3	3
126	56	11	6	2	2	1	1	3	0	1	0	1	2	0	3	0	1	2	3	1	3	3	0	0	1	1	2	2
127	78	11	7	2	2	1	2	0	1	3	0	1	2	0	0	1	2	3	0	2	3	3	1	1	2	2	0	0
128	81	11	8	2	2	1	3	1	3	0	0	1	2	0	1	2	3	0	1	3	3	3	2	2	0	0	1	1
129	34	11	9	2	2	2	0	2	2	2	1	3	0	2	2	3	1	3	0	2	0	1	0	1	0	1	0	1
130	45	11	10	2	2	2	1	3	0	1	1	3	0	2	3	0	2	0	1	3	0	1	1	2	2	3	3	0
131	45	11	11	2	2	2	2	0	1	3	1	3	0	2	0	1	3	1	2	0	0	1	2	3	3	0	1	2
132	54	11	12	2	2	2	3	1	3	0	1	3	0	2	1	2	0	2	3	1	0	1	3	0	1	2	2	3
133	54	12	1	2	3	0	0	3	3	3	2	2	1	1	2	3	2	2	1	1	3	2	3	2	3	2	3	2
134	54	12	2	2	3	0	1	2	1	0	2	2	1	1	3	0	3	3	2	2	3	2	2	1	1	0	0	0
135	12	12	3	2	3	0	2	1	0	2	2	2	1	1	0	1	0	0	3	3	3	2	1	0	0	0	2	1
136	15	12	4	2	3	0	3	0	2	1	2	2	1	1	1	2	1	1	0	0	3	2	0	0	2	1	1	0
137	45	12	5	2	3	1	0	3	3	3	0	1	2	0	2	3	3	1	2	0	0	0	0	0	0	0	0	0
138	46	12	6	2	3	1	1	2	1	0	0	1	2	0	3	0	1	2	3	1	0	0	3	3	2	2	1	1
139	56	12	7	2	3	1	2	1	0	2	0	1	2	0	0	1	2	3	0	2	0	0	2	2	1	1	3	3
140	65	12	8	2	3	1	3	0	2	1	0	1	2	0	1	2	3	0	1	3	0	0	1	1	3	3	2	2
141	66	12	9	2	3	2	0	3	3	3	1	3	0	2	2	3	1	3	0	2	1	2	1	2	1	2	1	2
142	46	12	10	2	3	2	1	2	1	0	1	3	0	2	3	0	2	0	1	3	1	2	0	1	3	0	2	3
143	47	12	11	2	3	2	2	1	0	2	1	3	0	2	0	1	3	1	2	0	1	2	3	0	2	3	0	1
144	48	12	12	2	3	2	3	0	2	1	1	3	0	2	1	2	0	2	3	1	1	2	2	3	0	1	3	0

Obs	F20	F21	F22	F23	F24	F25	F26	F27	F28	F29	F30	F31	F32	F33	F34	F35	F36	F37	F38	F39	F40	F41
121	0	0	3	3	0	1	0	1	0	1	0	3	3	3	0	3	3	3	0	0	3	3
122	0	0	3	3	1	2	2	3	3	0	1	1	0	0	2	1	3	2	3	3	2	3
123	0	0	3	3	2	3	3	0	1	2	2	1	3	2	3	2	2	3	1	1	0	0
124	0	0	3	3	3	0	1	2	2	3	3	2	2	3	1	1	0	0	2	3	3	2
125	2	3	0	2	0	1	0	1	0	1	2	3	0	2	2	3	0	2	2	3	0	2
126	2	3	0	2	1	2	2	3	3	0	3	0	1	3	3	3	2	0	1	2	3	1
127	2	3	0	2	2	3	3	0	1	2	3	3	2	0	1	2	3	1	3	0	1	3
128	2	3	0	2	3	0	1	2	2	3	1	2	3	1	3	0	1	3	3	1	2	0
129	3	1	2	0	0	1	0	1	0	1	3	1	2	0	3	1	2	0	3	1	2	0
130	3	1	2	0	1	2	2	3	3	0	0	2	3	1	1	3	0	2	2	0	1	3
131	3	1	2	0	2	3	3	0	1	2	1	3	0	2	2	0	1	3	0	2	3	1
132	3	1	2	0	3	0	1	2	2	3	2	0	1	3	0	2	3	1	1	3	0	2
133	1	1	0	0	1	2	1	2	1	2	1	1	0	0	1	1	0	0	1	1	0	0
134	1	1	0	0	0	1	3	0	2	3	0	3	3	3	3	2	2	3	2	3	3	2
135	1	1	0	0	3	0	2	3	0	1	3	2	2	3	2	1	3	2	0	0	3	3
136	1	1	0	0	2	3	0	1	3	0	2	1	3	2	0	3	3	3	3	3	2	3
137	3	0	1	3	1	2	1	2	1	2	3	0	1	3	3	0	1	3	3	0	1	3
138	3	0	1	3	0	1	3	0	2	3	2	3	0	2	1	2	3	1	3	1	2	0
139	3	0	1	3	3	0	2	3	0	1	1	2	3	1	3	3	2	0	2	3	0	2
140	3	0	1	3	2	3	0	1	3	0	3	3	2	0	2	3	0	2	1	2	3	1
141	0	2	3	1	1	2	1	2	1	2	0	2	3	1	0	2	3	1	0	2	3	1
142	0	2	3	1	0	1	3	0	2	3	3	1	2	0	2	0	1	3	1	3	0	2
143	0	2	3	1	3	0	2	3	0	1	2	0	1	3	1	3	0	2	3	1	2	0
144	0	2	3	1	2	3	0	1	3	0	1	3	0	2	3	1	2	0	2	0	1	3

Class Level Information

Class	Levels	Values
ROW	12	1 2 3 4 5 6 7 8 9 10 11 12
COL	12	1 2 3 4 5 6 7 8 9 10 11 12
A	3	0 1 2
B	4	0 1 2 3
C	3	0 1 2
D	4	0 1 2 3

Number of observations 144

Dependent Variable: Y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	143	56628.88889	396.00622	.	.
Error	0	0.00000	.		
Corrected Total	143	56628.88889			

R-Square	Coeff Var	Root MSE	Y Mean
1.000000	.	.	42.77778

Source	DF	Type I SS	Mean Square	F Value	Pr > F
ROW	11	22946.22222	2086.02020	.	.
COL	11	12564.88889	1142.26263	.	.
A*C	4	308.86111	77.21528	.	.
A*D	6	106.83333	17.80556	.	.
A*C*D	12	1514.25000	126.18750	.	.
B*C	6	2210.83333	368.47222	.	.
B*D	9	3344.00000	371.55556	.	.
B*C*D	18	6763.16667	375.73148	.	.
A*B*C	12	919.25000	76.60417	.	.
A*B*D	18	1781.83333	98.99074	.	.
A*B*C*D	36	4168.75000	115.79861	.	.

Source	DF	Type III SS	Mean Square	F Value	Pr > F
ROW	0	0.000000	.	.	.
COL	0	0.000000	.	.	.
A*C	4	308.861111	77.215278	.	.
A*D	6	106.833333	17.805556	.	.
A*C*D	12	1514.250000	126.187500	.	.
B*C	6	2210.833333	368.472222	.	.
B*D	9	3344.000000	371.555556	.	.
B*C*D	18	6763.166667	375.731481	.	.
A*B*C	12	919.250000	76.604167	.	.
A*B*D	18	1781.833333	98.990741	.	.
A*B*C*D	36	4168.750000	115.798611	.	.

Class Level Information

Class	Levels	Values
ROW	12	1 2 3 4 5 6 7 8 9 10 11 12
COL	12	1 2 3 4 5 6 7 8 9 10 11 12
A	3	0 1 2
B	4	0 1 2 3
C	3	0 1 2
D	4	0 1 2 3

F1	3	0 1 2
F2	4	0 1 2 3
F3	4	0 1 2 3
F4	4	0 1 2 3
F5	4	0 1 2 3
F6	4	0 1 2 3
F7	4	0 1 2 3
F8	4	0 1 2 3
F9	4	0 1 2 3
F10	4	0 1 2 3
F11	4	0 1 2 3
F12	4	0 1 2 3
F13	4	0 1 2 3
F14	4	0 1 2 3
F15	4	0 1 2 3
F16	4	0 1 2 3
F17	4	0 1 2 3
F18	4	0 1 2 3
F19	4	0 1 2 3
F20	4	0 1 2 3
F21	4	0 1 2 3
F22	4	0 1 2 3
F23	4	0 1 2 3
F24	4	0 1 2 3
F25	4	0 1 2 3
F26	4	0 1 2 3
F27	4	0 1 2 3
F28	4	0 1 2 3
F29	4	0 1 2 3
F30	4	0 1 2 3
F31	4	0 1 2 3
F32	4	0 1 2 3
F33	4	0 1 2 3
F34	4	0 1 2 3
F35	4	0 1 2 3
F36	4	0 1 2 3
F37	4	0 1 2 3
F38	4	0 1 2 3
F39	4	0 1 2 3
F40	4	0 1 2 3
F41	4	0 1 2 3

Number of observations 144

Dependent Variable: Y Sum of

Source	DF	Squares	Mean Square	F Value	Pr > F
Model	143	56628.88889	396.00622	.	.
Error	0	0.00000	.		
Corrected Total	143	56628.88889			

R-Square	Coeff Var	Root MSE	Y Mean
1.000000	.	.	42.77778

Source	DF	Type I SS	Mean Square	F Value	Pr > F
ROW	11	22946.22222	2086.02020	.	.
COL	11	12564.88889	1142.26263	.	.
F1	2	154.43056	77.21528	.	.
F2	2	154.43056	77.21528	.	.
F3	3	55.75000	18.58333	.	.
F4	3	51.08333	17.02778	.	.
F5	3	345.28154	115.09385	.	.
F6	3	407.97023	135.99008	.	.
F7	3	447.42553	149.14184	.	.
F8	3	313.57270	104.52423	.	.
F9	3	405.62500	135.20833	.	.
F10	3	1805.20833	601.73611	.	.
F11	3	291.00000	97.00000	.	.
F12	3	1663.33333	554.44444	.	.
F13	3	1389.66667	463.22222	.	.
F14	3	2283.79167	761.26389	.	.
F15	3	2009.70833	669.90278	.	.
F16	3	182.29167	60.76389	.	.
F17	3	482.20833	160.73611	.	.
F18	3	905.95833	301.98611	.	.
F19	3	899.20833	299.73611	.	.
F20	3	66.57865	22.19288	.	.
F21	3	323.14712	107.71571	.	.
F22	3	49.32638	16.44213	.	.
F23	3	480.19784	160.06595	.	.
F24	3	123.41667	41.13889	.	.
F25	3	154.08333	51.36111	.	.
F26	3	243.25000	81.08333	.	.
F27	3	527.58333	175.86111	.	.
F28	3	432.91667	144.30556	.	.
F29	3	300.58333	100.19444	.	.
F30	3	785.96516	261.98839	.	.
F31	3	169.01885	56.33962	.	.
F32	3	713.47108	237.82369	.	.
F33	3	251.79490	83.93163	.	.

F34	3	123.06900	41.02300	.	.
F35	3	137.31196	45.77065	.	.
F36	3	314.31562	104.77187	.	.
F37	3	365.88674	121.96225	.	.
F38	3	399.59416	133.19805	.	.
F39	3	797.08659	265.69553	.	
F40	3	44.43038	14.81013	.	.
F41	3	66.80553	22.26851	.	.

Source	DF	Type III SS	Mean Square	F Value	Pr > F
ROW	10	18139.91570	1813.99157	.	.
COL	10	5529.56213	552.95621	.	.
F1	1	48.79407	48.79407	.	.
F2	2	51.78828	25.89414	.	.
F3	3	46.58723	15.52908	.	.
F4	3	28.49455	9.49818	.	.
F5	3	462.79937	154.26646	.	.
F6	3	550.79526	183.59842	.	.
F7	3	246.74345	82.24782	.	.
F8	3	313.57270	104.52423	.	.
F9	3	1340.46906	446.82302	.	.
F10	3	1706.92707	568.97569	.	.
F11	3	162.45226	54.15075	.	.
F12	3	152.03512	50.67837	.	.
F13	3	1528.87265	509.62422	.	.
F14	3	935.20225	311.73408	.	.
F15	3	1075.91914	358.63971	.	.
F16	3	163.04439	54.34813	.	.
F17	3	236.25319	78.75106	.	.
F18	3	338.16288	112.72096	.	.
F19	3	736.22970	245.40990	.	.
F20	3	303.03299	101.01100	.	.
F21	3	445.36862	148.45621	.	.
F22	3	204.45424	68.15141	.	.
F23	3	480.19784	160.06595	.	.
F24	3	65.06948	21.68983	.	.
F25	3	96.15809	32.05270	.	.
F26	3	531.91954	177.30651	.	.
F27	3	548.54145	182.84715	.	.
F28	3	454.62130	151.54043	.	.
F29	3	87.84470	29.28157	.	
F30	3	189.11845	63.03948	.	
F31	3	890.77558	296.92519	.	.
F32	3	635.94172	211.98057	.	.
F33	3	251.79490	83.93163	.	.

F34	3	97.85601	32.61867	.	.
F35	3	309.59416	103.19805	.	.
F36	3	291.66728	97.22243	.	.
F37	3	365.88674	121.96225	.	.
F38	3	50.82684	16.94228	.	.
F39	3	230.50693	76.83564	.	.
F40	3	61.16278	20.38759	.	.
F41	3	66.80553	22.26851	.	.

4. COMPLETE SET OF SSOFSs FOR $n = 2 \times 6$

Let the row numbers 1 to 12 correspond to the 12 combinations of factor A at two levels (0, 1) and of factor B at six levels (0, 1, 2, 3, 4, 5). Let the 12 column numbers 1 to 12 correspond to the 12 combinations of factor C at two levels (0, 1) and factor D at six levels (0, 1, 2, 3, 4, 5). In the previous two sections, a MOLS(4, 3) set was used. A similar set is needed for the B × D interaction for the number six. Since no MOLS(6, 5) set exists, something else is required. Consider the following set of five 6 × 6 Latin squares:

L1	L2	L3	L4	L5
0 1 2 3 4 5	0 1 2 3 4 5	0 1 2 3 4 5	0 1 2 3 4 5	0 1 2 3 4 5
1 2 0 5 3 4	5 3 4 2 0 1	4 5 3 1 2 0	3 4 5 0 1 2	2 0 1 4 5 3
2 0 1 4 5 3	1 2 0 5 3 4	5 3 4 2 0 1	4 5 3 1 2 0	3 4 5 0 1 2
3 4 5 0 1 2	2 0 1 4 5 3	1 2 0 5 3 4	5 3 4 2 0 1	4 5 3 1 2 0
4 5 3 1 2 0	3 4 5 0 1 2	2 0 1 4 5 3	1 2 0 5 3 4	5 3 4 2 0 1
5 3 4 2 0 1	4 5 3 1 2 0	3 4 5 0 1 2	2 0 1 4 5 3	1 2 0 5 3 4

Square L1 is a form of a Kronecker product of a 2 × 2 and a 3 × 3 Latin square. Square L2 is formed by moving the last row of L1 to the second row and pushing all the other rows of L1 down one row. This is a cyclical permutation of the last five rows of L1. L3, L4, and L5 are similarly formed by cyclical permutations of the rows of the preceding square. Given a set of 36 observations in a 6 × 6 square, let the rows correspond to the factor B and the columns to the factor D. Then the row × column interaction is equal to the B × D interaction. Sums of squares for the above five arrangements for these 36 observations add to that for the interaction B × D. Thus these five Latin squares are sum of squares orthogonal. We denote this set as a sum of squares orthogonal set of Latin squares of side six, SSOLS(6, 5).

Using this SSOLS(6, 5) and entering it in the data set as F5, F6, F7, F8, and F9, a SAS code was written to construct the remaining F-squares to make a complete set of sum of squares orthogonal F-squares. The SAS code and data set is given in Table 4.1. There are 25 F-squares needed to complete the set. Square F1 has two symbols (0, 1) and the remaining has six symbols (0, 1, 2, 3, 4, 5). A series of "IF" and "THEN" statements are used to obtain modulus two for F1 and modulus six for the remaining F-squares. It may be possible to write a code that would also construct F5, F6, F7, F8, and F9 rather than inserting them in the data set as is done in Table 4.1

.

Table 4.1. SAS code and analyses of variance for a complete set of SSOFSs for $n = 2 \times 6$.

```
data FSS2626;
input Y ROW COL A B C D  F5 F6 F7 F8 F9 ;

F1 = A + C; F2=A+D;  F3=F1+D; F4=B+C; F10= F5+C; F11=F6+C;
F12= F7+C; F13=F8+C; F14=F9+C; F15=B+F1; F16=F5+A;
F17=F6+A; F18=F7+A; F19=F8+A; F20=F9+A; F21=F1+F5;
F22=F1+F6; F23=F1+F7; F24= F1+F8; F25=F1+F9;

IF F1>1 THEN F1=A+C-2; IF F2>5 THEN F2=A+D-6;
IF F3>5 THEN F3=F1+D-6; IF F3<0 THEN F3=F1+D;
IF F4>5 THEN F4=B+C-6;
IF F10>5 THEN F10=F5+C-6; IF F11>5 THEN F11=F6+C-6;
IF F12>5 THEN F12=F7+C-6; IF F13>5 THEN F13=F8+C-6;
IF F14>5 THEN F14=F9+C-6;
IF F15>5 THEN F15=B+F1-6; IF F15<0 THEN F15= B+F1;
IF F16>5 THEN F16=F5+A-6; IF F17>5 THEN F17= F6+A-6;
IF F18>5 THEN F18=F7+A-6; IF F19>5 THEN F19=F8+A-6;
IF F20>5 THEN F20=F9+A-6;
IF F21>5 THEN F21=F1+F5-6; IF F21<0 THEN F21= F1+F5;
IF F22>5 THEN F22=F1+F6-6; IF F22<0 THEN F22= F1+F6;
IF F23>5 THEN F23=F1+F7-6; IF F23<0 THEN F23= F1+F7;
IF F24>5 THEN F24=F1+F8-6; IF F24<0 THEN F24= F1+F8;
IF F25>5 THEN F25=F1+F9-6; IF F25<0 THEN F25= F1+F9;

DATALINES;  /* 5 6 7 8 9*/
10 1 1 0 0 0 0  0 0 0 0 0
11 1 2 0 0 0 1  1 1 1 1 1
12 1 3 0 0 0 2  2 2 2 2 2
13 1 4 0 0 0 3  3 3 3 3 3
14 1 5 0 0 0 4  4 4 4 4 4
15 1 6 0 0 0 5  5 5 5 5 5
16 1 7 0 0 1 0  0 0 0 0 0
17 1 8 0 0 1 1  1 1 1 1 1
18 1 9 0 0 1 2  2 2 2 2 2
18 1 10 0 0 1 3  3 3 3 3 3
18 1 11 0 0 1 4  4 4 4 4 4
18 1 12 0 0 1 5  5 5 5 5 5
19 2 1 0 1 0 0  1 5 4 3 2
20 2 2 0 1 0 1  2 3 5 4 0
25 2 3 0 1 0 2  0 4 3 5 1
99 2 4 0 1 0 3  5 2 1 0 4
55 2 5 0 1 0 4  3 0 2 1 5
55 2 6 0 1 0 5  4 1 0 2 3
55 2 7 0 1 1 0  1 5 4 3 2
45 2 8 0 1 1 1  2 3 5 4 0
46 2 9 0 1 1 2  0 4 3 5 1
```

47 2 10 0 1 1 3 5 2 1 0 4
47 2 11 0 1 1 4 3 0 2 1 5
47 2 12 0 1 1 5 4 1 0 2 3
47 3 1 0 2 0 0 2 1 5 4 3
31 3 2 0 2 0 1 0 2 3 5 4
33 3 3 0 2 0 2 1 0 4 3 5
56 3 4 0 2 0 3 4 5 2 1 0
56 3 5 0 2 0 4 5 3 0 2 1
56 3 6 0 2 0 5 3 4 1 0 2
78 3 7 0 2 1 0 2 1 5 4 3
81 3 8 0 2 1 1 0 2 3 5 4
34 3 9 0 2 1 2 1 0 4 3 5
45 3 10 0 2 1 3 4 5 2 1 0
45 3 11 0 2 1 4 5 3 0 2 1
54 3 12 0 2 1 5 3 4 1 0 2
54 4 1 0 3 0 0 3 2 1 5 4
54 4 2 0 3 0 1 4 0 2 3 5
12 4 3 0 3 0 2 5 1 0 4 3
15 4 4 0 3 0 3 0 4 5 2 1
45 4 5 0 3 0 4 1 5 3 0 2
46 4 6 0 3 0 5 2 3 4 1 0
56 4 7 0 3 1 0 3 2 1 5 4
65 4 8 0 3 1 1 4 0 2 3 5
66 4 9 0 3 1 2 5 1 0 4 3
46 4 10 0 3 1 3 0 4 5 2 1
47 4 11 0 3 1 4 1 5 3 0 2
48 4 12 0 3 1 5 2 3 4 1 0
10 5 1 0 4 0 0 4 3 2 1 5
11 5 2 0 4 0 1 5 4 0 2 3
12 5 3 0 4 0 2 3 5 1 0 4
13 5 4 0 4 0 3 1 0 4 5 2
14 5 5 0 4 0 4 2 1 5 3 0
15 5 6 0 4 0 5 0 2 3 4 1
16 5 7 0 4 1 0 4 3 2 1 5
17 5 8 0 4 1 1 5 4 0 2 3
18 5 9 0 4 1 2 3 5 1 0 4
18 5 10 0 4 1 3 1 0 4 5 2
18 5 11 0 4 1 4 2 1 5 3 0
18 5 12 0 4 1 5 0 2 3 4 1
19 6 1 0 5 0 0 5 4 3 2 1
20 6 2 0 5 0 1 3 5 4 0 2
25 6 3 0 5 0 2 4 3 5 1 0
99 6 4 0 5 0 3 2 1 0 4 5
55 6 5 0 5 0 4 0 2 1 5 3
55 6 6 0 5 0 5 1 0 2 3 4
55 6 7 0 5 1 0 5 4 3 2 1

45 6 8 0 5 1 1 3 5 4 0 2
46 6 9 0 5 1 2 4 3 5 1 0
47 6 10 0 5 1 3 2 1 0 4 5
47 6 11 0 5 1 4 0 2 1 5 3
47 6 12 0 5 1 5 1 0 2 3 4
47 7 1 1 0 0 0 0 0 0 0 0
31 7 2 1 0 0 1 1 1 1 1 1
33 7 3 1 0 0 2 2 2 2 2 2
56 7 4 1 0 0 3 3 3 3 3 3
56 7 5 1 0 0 4 4 4 4 4 4
56 7 6 1 0 0 5 5 5 5 5 5
78 7 7 1 0 1 0 0 0 0 0 0
81 7 8 1 0 1 1 1 1 1 1 1
34 7 9 1 0 1 2 2 2 2 2 2
45 7 10 1 0 1 3 3 3 3 3 3
45 7 11 1 0 1 4 4 4 4 4 4
54 7 12 1 0 1 5 5 5 5 5 5
54 8 1 1 1 0 0 1 5 4 3 2
54 8 2 1 1 0 1 2 3 5 4 0
12 8 3 1 1 0 2 0 4 3 5 1
15 8 4 1 1 0 3 5 2 1 0 4
45 8 5 1 1 0 4 3 0 2 1 5
46 8 6 1 1 0 5 4 1 0 2 3
56 8 7 1 1 1 0 1 5 4 3 2
65 8 8 1 1 1 1 2 3 5 4 0
66 8 9 1 1 1 2 0 4 3 5 1
46 8 10 1 1 1 3 5 2 1 0 4
47 8 11 1 1 1 4 3 0 2 1 5
48 8 12 1 1 1 5 4 1 0 2 3
47 9 1 1 2 0 0 2 1 5 4 3
31 9 2 1 2 0 1 0 2 3 5 4
33 9 3 1 2 0 2 1 0 4 3 5
56 9 4 1 2 0 3 4 5 2 1 0
56 9 5 1 2 0 4 5 3 0 2 1
56 9 6 1 2 0 5 3 4 1 0 2
78 9 7 1 2 1 0 2 1 5 4 3
81 9 8 1 2 1 1 0 2 3 5 4
34 9 9 1 2 1 2 1 0 4 3 5
45 9 10 1 2 1 3 4 5 2 1 0
45 9 11 1 2 1 4 5 3 0 2 1
54 9 12 1 2 1 5 3 4 1 0 2
54 10 1 1 3 0 0 3 2 1 5 4
54 10 2 1 3 0 1 4 0 2 3 5
12 10 3 1 3 0 2 5 1 0 4 3
15 10 4 1 3 0 3 0 4 5 2 1
45 10 5 1 3 0 4 1 5 3 0 2

```
46 10 6 1 3 0 5  2 3 4 1 0
56 10 7 1 3 1 0  3 2 1 5 4
65 10 8 1 3 1 1  4 0 2 3 5
66 10 9 1 3 1 2  5 1 0 4 3
46 10 10 1 3 1 3 0 4 5 2 1
47 10 11 1 3 1 4 1 5 3 0 2
48 10 12 1 3 1 5 2 3 4 1 0
47 11 1 1 4 0 0  4 3 2 1 5
31 11 2 1 4 0 1  5 4 0 2 3
33 11 3 1 4 0 2  3 5 1 0 4
56 11 4 1 4 0 3  1 0 4 5 2
56 11 5 1 4 0 4  2 1 5 3 0
56 11 6 1 4 0 5  0 2 3 4 1
78 11 7 1 4 1 0  4 3 2 1 5
81 11 8 1 4 1 1  5 4 0 2 3
34 11 9 1 4 1 2  3 5 1 0 4
45 11 10 1 4 1 3 1 0 4 5 2
45 11 11 1 4 1 4 2 1 5 3 0
54 11 12 1 4 1 5 0 2 3 4 1
54 12 1 1 5 0 0  5 4 3 2 1
54 12 2 1 5 0 1  3 5 4 0 2
12 12 3 1 5 0 2  4 3 5 1 0
15 12 4 1 5 0 3  2 1 0 4 5
45 12 5 1 5 0 4  0 2 1 5 3
46 12 6 1 5 0 5  1 0 2 3 4
56 12 7 1 5 1 0  5 4 3 2 1
65 12 8 1 5 1 1  3 5 4 0 2
66 12 9 1 5 1 2  4 3 5 1 0
46 12 10 1 5 1 3 2 1 0 4 5
47 12 11 1 5 1 4 0 2 1 5 3
48 12 12 1 5 1 5 1 0 2 3 4
; RUN;PROC PRINT;
PROC GLM DATA = FSS2626;

CLASS ROW COL A B C D;

MODEL Y = ROW COL A*C A*D A*C*D B*C B*D B*C*D A*B*C A*B*D A*B*C*D;

RUN;

PROC GLM DATA = FSS2626;

CLASS ROW COL A B C D F1 F2 F3 F4 F5 F6 F7 F8 F9 F10 F11 F12 F13
F14 F15 F16 F17 F18 F19 F20 F21 F22 F23 F24 F25;

MODEL Y = ROW COL F1 F2 F3 F4 F5 F6 F7 F8 F9 F10 F11 F12 F13
```

F14 F15 F16 F17 F18 F19 F20 F21 F22 F23 F24 F25;

RUN;

The output for the above code and data set is given in Table 4.2. The first column is the observation number (1 to 144), the second column is the response Y, the third column is the row number, the fourth column is the column number, the fifth column is the level for factor A (0, 1), the sixth column is the level of factor B, the seventh column is the level of factor C, the eighth column is the level of factor D, and the next 25 columns are the F-square symbols in the order indicated in the table.

Table 4.2. SAS output for the example in Table 4.1.

```
O    R C                    F F F F F F F F F F F F F F F F
b    O O          F F F F F F F F F1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2
s  Y W L A B C D 5 6 7 8 9 1 2 3 4  0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5

 1 10 1  1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 2 11 1  2 0 0 0 1 1 1 1 1 1 0 1 1 0 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1
 3 12 1  3 0 0 0 2 2 2 2 2 2 0 2 2 0 2 2 2 2 2 0 2 2 2 2 2 2 2 2 2
 4 13 1  4 0 0 0 3 3 3 3 3 3 0 3 3 0 3 3 3 3 3 0 3 3 3 3 3 3 3 3 3
 5 14 1  5 0 0 0 4 4 4 4 4 4 0 4 4 0 4 4 4 4 4 0 4 4 4 4 4 4 4 4 4
 6 15 1  6 0 0 0 5 5 5 5 5 5 0 5 5 0 5 5 5 5 5 0 5 5 5 5 5 5 5 5 5
 7 16 1  7 0 0 1 0 0 0 0 0 0 1 0 1 1 1 1 1 1 1 1 0 0 0 0 0 1 1 1 1 1
 8 17 1  8 0 0 1 1 1 1 1 1 1 1 2 1 2 2 2 2 2 1 1 1 1 1 1 2 2 2 2 2
 9 18 1  9 0 0 1 2 2 2 2 2 2 1 2 3 1 3 3 3 3 3 1 2 2 2 2 2 3 3 3 3 3
10 18 1 10 0 0 1 3 3 3 3 3 3 1 3 4 1 4 4 4 4 4 1 3 3 3 3 3 4 4 4 4 4
11 18 1 11 0 0 1 4 4 4 4 4 4 1 4 5 1 5 5 5 5 5 1 4 4 4 4 4 5 5 5 5 5
12 18 1 12 0 0 1 5 5 5 5 5 5 1 5 0 1 0 0 0 0 0 1 5 5 5 5 5 0 0 0 0 0
13 19 2  1 0 1 0 0 1 5 4 3 2 0 0 0 1 1 5 4 3 2 1 1 5 4 3 2 1 5 4 3 2
14 20 2  2 0 1 0 1 2 3 5 4 0 0 1 1 1 2 3 5 4 0 1 2 3 5 4 0 2 3 5 4 0
15 25 2  3 0 1 0 2 0 4 3 5 1 0 2 2 1 0 4 3 5 1 1 0 4 3 5 1 0 4 3 5 1
16 99 2  4 0 1 0 3 5 2 1 0 4 0 3 3 1 5 2 1 0 4 1 5 2 1 0 4 5 2 1 0 4
17 55 2  5 0 1 0 4 3 0 2 1 5 0 4 4 1 3 0 2 1 5 1 3 0 2 1 5 3 0 2 1 5
18 55 2  6 0 1 0 5 4 1 0 2 3 0 5 5 1 4 1 0 2 3 1 4 1 0 2 3 4 1 0 2 3
19 55 2  7 0 1 1 0 1 5 4 3 2 1 0 1 2 2 0 5 4 3 2 1 5 4 3 2 2 0 5 4 3
20 45 2  8 0 1 1 1 2 3 5 4 0 1 1 2 2 3 4 0 5 1 2 2 3 5 4 0 3 4 0 5 1
21 46 2  9 0 1 1 2 0 4 3 5 1 1 2 3 2 1 5 4 0 2 2 0 4 3 5 1 1 5 4 0 2
22 47 2 10 0 1 1 3 5 2 1 0 4 1 3 4 2 0 3 2 1 5 2 5 2 1 0 4 0 3 2 1 5
23 47 2 11 0 1 1 4 3 0 2 1 5 1 4 5 2 4 1 3 2 0 2 3 0 2 1 5 4 1 3 2 0
24 47 2 12 0 1 1 5 4 1 0 2 3 1 5 0 2 5 2 1 3 4 2 4 1 0 2 3 5 2 1 3 4
25 47 3  1 0 2 0 0 2 1 5 4 3 0 0 0 2 2 1 5 4 3 2 2 1 5 4 3 2 1 5 4 3
26 31 3  2 0 2 0 1 0 2 3 5 4 0 1 1 2 0 2 3 5 4 2 0 2 3 5 4 0 2 3 5 4
27 33 3  3 0 2 0 2 1 0 4 3 5 0 2 2 2 1 0 4 3 5 2 1 0 4 3 5 1 0 4 3 5
28 56 3  4 0 2 0 3 4 5 2 1 0 0 3 3 2 4 5 2 1 0 2 4 5 2 1 0 4 5 2 1 0
29 56 3  5 0 2 0 4 5 3 0 2 1 0 4 4 2 5 3 0 2 1 2 5 3 0 2 1 5 3 0 2 1
```

30 56 3 6 0 2 0 5 3 4 1 0 2 0 5 5 2 3 4 1 0 2 2 3 4 1 0 2 3 4 1 0 2
31 78 3 7 0 2 1 0 2 1 5 4 3 1 0 1 3 3 2 0 5 4 3 2 1 5 4 3 3 2 0 5 4
32 81 3 8 0 2 1 1 0 2 3 5 4 1 1 2 3 1 3 4 0 5 3 0 2 3 5 4 1 3 4 0 5
33 34 3 9 0 2 1 2 1 0 4 3 5 1 2 3 3 2 1 5 4 0 3 1 0 4 3 5 2 1 5 4 0
34 45 3 10 0 2 1 3 4 5 2 1 0 1 3 4 3 5 0 3 2 1 3 4 5 2 1 0 5 0 3 2 1
35 45 3 11 0 2 1 4 5 3 0 2 1 1 4 5 3 0 4 1 3 2 3 5 3 0 2 1 0 4 1 3 2
36 54 3 12 0 2 1 5 3 4 1 0 2 1 5 0 3 4 5 2 1 3 3 3 4 1 0 2 4 5 2 1 3
37 54 4 1 0 3 0 0 3 2 1 5 4 0 0 0 3 3 2 1 5 4 3 3 2 1 5 4 3 2 1 5 4
38 54 4 2 0 3 0 1 4 0 2 3 5 0 1 1 3 4 0 2 3 5 3 4 0 2 3 5 4 0 2 3 5
39 12 4 3 0 3 0 2 5 1 0 4 3 0 2 2 3 5 1 0 4 3 3 5 1 0 4 3 5 1 0 4 3
40 15 4 4 0 3 0 3 0 4 5 2 1 0 3 3 3 0 4 5 2 1 3 0 4 5 2 1 0 4 5 2 1
41 45 4 5 0 3 0 4 1 5 3 0 2 0 4 4 3 1 5 3 0 2 3 1 5 3 0 2 1 5 3 0 2
42 46 4 6 0 3 0 5 2 3 4 1 0 0 5 5 3 2 3 4 1 0 3 2 3 4 1 0 2 3 4 1 0
43 56 4 7 0 3 1 0 3 2 1 5 4 1 0 1 4 4 3 2 0 5 4 3 2 1 5 4 4 3 2 0 5
44 65 4 8 0 3 1 1 4 0 2 3 5 1 1 2 4 5 1 3 4 0 4 4 0 2 3 5 5 1 3 4 0
45 66 4 9 0 3 1 2 5 1 0 4 3 1 2 3 4 0 2 1 5 4 4 5 1 0 4 3 0 2 1 5 4
46 46 4 10 0 3 1 3 0 4 5 2 1 1 3 4 4 1 5 0 3 2 4 0 4 5 2 1 1 5 0 3 2
47 47 4 11 0 3 1 4 1 5 3 0 2 1 4 5 4 2 0 4 1 3 4 1 5 3 0 2 2 0 4 1 3
48 48 4 12 0 3 1 5 2 3 4 1 0 1 5 0 4 3 4 5 2 1 4 2 3 4 1 0 3 4 5 2 1
49 10 5 1 0 4 0 0 4 3 2 1 5 0 0 0 4 4 3 2 1 5 4 4 3 2 1 5 4 3 2 1 5
50 11 5 2 0 4 0 1 5 4 0 2 3 0 1 1 4 5 4 0 2 3 4 5 4 0 2 3 5 4 0 2 3
51 12 5 3 0 4 0 2 3 5 1 0 4 0 2 2 4 3 5 1 0 4 4 3 5 1 0 4 3 5 1 0 4
52 13 5 4 0 4 0 3 1 0 4 5 2 0 3 3 4 1 0 4 5 2 4 1 0 4 5 2 1 0 4 5 2
53 14 5 5 0 4 0 4 2 1 5 3 0 0 4 4 4 2 1 5 3 0 4 2 1 5 3 0 2 1 5 3 0
54 15 5 6 0 4 0 5 0 2 3 4 1 0 5 5 4 0 2 3 4 1 4 0 2 3 4 1 0 2 3 4 1
55 16 5 7 0 4 1 0 4 3 2 1 5 1 0 1 5 5 4 3 2 0 5 4 3 2 1 5 5 4 3 2 0
56 17 5 8 0 4 1 1 5 4 0 2 3 1 1 2 5 0 5 1 3 4 5 5 4 0 2 3 0 5 1 3 4
57 18 5 9 0 4 1 2 3 5 1 0 4 1 2 3 5 4 0 2 1 5 5 3 5 1 0 4 4 0 2 1 5
58 18 5 10 0 4 1 3 1 0 4 5 2 1 3 4 5 2 1 5 0 3 5 1 0 4 5 2 2 1 5 0 3
59 18 5 11 0 4 1 4 2 1 5 3 0 1 4 5 5 3 2 0 4 1 5 2 1 5 3 0 3 2 0 4 1
60 18 5 12 0 4 1 5 0 2 3 4 1 1 5 0 5 1 3 4 5 2 5 0 2 3 4 1 1 3 4 5 2
61 19 6 1 0 5 0 0 5 4 3 2 1 0 0 0 5 5 4 3 2 1 5 5 4 3 2 1 5 4 3 2 1
62 20 6 2 0 5 0 1 3 5 4 0 2 0 1 1 5 3 5 4 0 2 5 3 5 4 0 2 3 5 4 0 2
63 25 6 3 0 5 0 2 4 3 5 1 0 0 2 2 5 4 3 5 1 0 5 4 3 5 1 0 4 3 5 1 0
64 99 6 4 0 5 0 3 2 1 0 4 5 0 3 3 5 2 1 0 4 5 5 2 1 0 4 5 2 1 0 4 5
65 55 6 5 0 5 0 4 0 2 1 5 3 0 4 4 5 0 2 1 5 3 5 0 2 1 5 3 0 2 1 5 3
66 55 6 6 0 5 0 5 1 0 2 3 4 0 5 5 5 1 0 2 3 4 5 1 0 2 3 4 1 0 2 3 4
67 55 6 7 0 5 1 0 5 4 3 2 1 1 0 1 0 0 5 4 3 2 0 5 4 3 2 1 0 5 4 3 2
68 45 6 8 0 5 1 1 3 5 4 0 2 1 1 2 0 4 0 5 1 3 0 3 5 4 0 2 4 0 5 1 3
69 46 6 9 0 5 1 2 4 3 5 1 0 1 2 3 0 5 4 0 2 1 0 4 3 5 1 0 5 4 0 2 1
70 47 6 10 0 5 1 3 2 1 0 4 5 1 3 4 0 3 2 1 5 0 0 2 1 0 4 5 3 2 1 5 0
71 47 6 11 0 5 1 4 0 2 1 5 3 1 4 5 0 1 3 2 0 4 0 0 2 1 5 3 1 3 2 0 4
72 47 6 12 0 5 1 5 1 0 2 3 4 1 5 0 0 2 1 3 4 5 0 1 0 2 3 4 2 1 3 4 5
73 47 7 1 1 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1
74 31 7 2 1 0 0 1 1 1 1 1 1 1 2 2 0 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2
75 33 7 3 1 0 0 2 2 2 2 2 2 1 3 3 0 2 2 2 2 2 2 1 3 3 3 3 3 3 3 3 3 3

76 56 7 4 1 0 0 3 3 3 3 3 3 1 4 4 0 3 3 3 3 3 1 4 4 4 4 4 4 4 4 4
77 56 7 5 1 0 0 4 4 4 4 4 4 1 5 5 0 4 4 4 4 4 1 5 5 5 5 5 5 5 5 5
78 56 7 6 1 0 0 5 5 5 5 5 5 1 0 0 0 5 5 5 5 5 1 0 0 0 0 0 0 0 0 0
79 78 7 7 1 0 1 0 0 0 0 0 0 1 2 1 1 1 1 1 1 2 1 1 1 1 1 2 2 2 2 2
80 81 7 8 1 0 1 1 1 1 1 1 1 0 2 3 1 2 2 2 2 2 2 2 2 2 2 3 3 3 3 3
81 34 7 9 1 0 1 2 2 2 2 2 2 0 3 4 1 3 3 3 3 3 2 3 3 3 3 3 4 4 4 4 4
82 45 7 10 1 0 1 3 3 3 3 3 3 0 4 5 1 4 4 4 4 4 2 4 4 4 4 4 5 5 5 5 5
83 45 7 11 1 0 1 4 4 4 4 4 4 0 5 4 1 5 5 5 5 5 2 5 5 5 5 5 4 4 4 4 4
84 54 7 12 1 0 1 5 5 5 5 5 5 0 0 5 1 0 0 0 0 0 2 0 0 0 0 0 5 5 5 5 5
85 54 8 1 1 1 0 0 1 5 4 3 2 1 1 1 1 1 5 4 3 2 2 2 0 5 4 3 2 0 5 4 3
86 54 8 2 1 1 0 1 2 3 5 4 0 1 2 2 1 2 3 5 4 0 2 3 4 0 5 1 3 4 0 5 1
87 12 8 3 1 1 0 2 0 4 3 5 1 1 3 3 1 0 4 3 5 1 2 1 5 4 0 2 1 5 4 0 2
88 15 8 4 1 1 0 3 5 2 1 0 4 1 4 4 1 5 2 1 0 4 2 0 3 2 1 5 0 3 2 1 5
89 45 8 5 1 1 0 4 3 0 2 1 5 1 5 5 1 3 0 2 1 5 2 4 1 3 2 0 4 1 3 2 0
90 46 8 6 1 1 0 5 4 1 0 2 3 1 0 0 1 4 1 0 2 3 2 5 2 1 3 4 5 2 1 3 4
91 56 8 7 1 1 1 0 1 5 4 3 2 0 1 2 2 2 0 5 4 3 3 2 0 5 4 3 3 5 4 5 4
92 65 8 8 1 1 1 1 2 3 5 4 0 0 2 3 2 3 4 0 5 1 3 3 4 0 5 1 4 5 5 4 2
93 66 8 9 1 1 1 2 0 4 3 5 1 0 3 4 2 1 5 4 0 2 3 1 5 4 0 2 2 4 5 5 3
94 46 8 10 1 1 1 3 5 2 1 0 4 0 4 5 2 0 3 2 1 5 3 0 3 2 1 5 5 4 3 2 4
95 47 8 11 1 1 1 4 3 0 2 1 5 0 5 4 2 4 1 3 2 0 3 4 1 3 2 0 5 2 4 3 5
96 48 8 12 1 1 1 5 4 1 0 2 3 0 0 5 2 5 2 1 3 4 3 5 2 1 3 4 4 3 2 4 5
97 47 9 1 1 2 0 0 2 1 5 4 3 1 1 1 2 2 1 5 4 3 3 3 2 0 5 4 3 2 0 5 4
98 31 9 2 1 2 0 1 0 2 3 5 4 1 2 2 2 0 2 3 5 4 3 1 3 4 0 5 1 3 4 0 5
99 33 9 3 1 2 0 2 1 0 4 3 5 1 3 3 2 1 0 4 3 5 3 2 1 5 4 0 2 1 5 4 0
100 56 9 4 1 2 0 3 4 5 2 1 0 1 4 4 2 4 5 2 1 0 3 5 0 3 2 1 5 0 3 2 1
101 56 9 5 1 2 0 4 5 3 0 2 1 1 5 5 2 5 3 0 2 1 3 0 4 1 3 2 0 4 1 3 2
102 56 9 6 1 2 0 5 3 4 1 0 2 1 0 0 2 3 4 1 0 2 3 4 5 2 1 3 4 5 2 1 3
103 78 9 7 1 2 1 0 2 1 5 4 3 0 1 2 3 3 2 0 5 4 4 3 2 0 5 4 4 3 5 4 5
104 81 9 8 1 2 1 1 0 2 3 5 4 0 2 3 3 1 3 4 0 5 4 1 3 4 0 5 2 4 5 5 4
105 34 9 9 1 2 1 2 1 0 4 3 5 0 3 4 3 2 1 5 4 0 4 2 1 5 4 0 3 2 4 5 5
106 45 9 10 1 2 1 3 4 5 2 1 0 0 4 5 3 5 0 3 2 1 4 5 0 3 2 1 4 5 4 3 2
107 45 9 11 1 2 1 4 5 3 0 2 1 0 5 4 3 0 4 1 3 2 4 0 4 1 3 2 5 5 2 4 3
108 54 9 12 1 2 1 5 3 4 1 0 2 0 0 5 3 4 5 2 1 3 4 4 5 2 1 3 5 4 3 2 4
109 54 10 1 1 3 0 0 3 2 1 5 4 1 1 1 3 3 2 1 5 4 4 4 3 2 0 5 4 3 2 0 5
110 54 10 2 1 3 0 1 4 0 2 3 5 1 2 2 3 4 0 2 3 5 4 5 1 3 4 0 5 1 3 4 0
111 12 10 3 1 3 0 2 5 1 0 4 3 1 3 3 3 5 1 0 4 3 4 0 2 1 5 4 0 2 1 5 4
112 15 10 4 1 3 0 3 0 4 5 2 1 1 4 4 3 0 4 5 2 1 4 1 5 0 3 2 1 5 0 3 2
113 45 10 5 1 3 0 4 1 5 3 0 2 1 5 5 3 1 5 3 0 2 4 2 0 4 1 3 2 0 4 1 3
114 46 10 6 1 3 0 5 2 3 4 1 0 1 0 0 3 2 3 4 1 0 4 3 4 5 2 1 3 4 5 2 1
115 56 10 7 1 3 1 0 3 2 1 5 4 0 1 2 4 4 3 2 0 5 5 4 3 2 0 5 5 4 3 5 4
116 65 10 8 1 3 1 1 4 0 2 3 5 0 2 3 4 5 1 3 4 0 5 5 1 3 4 0 4 2 4 5 5
117 66 10 9 1 3 1 2 5 1 0 4 3 0 3 4 4 0 2 1 5 4 5 0 2 1 5 4 5 3 2 4 5
118 46 10 10 1 3 1 3 0 4 5 2 1 0 4 5 4 1 5 0 3 2 5 1 5 0 3 2 2 4 5 4 3
119 47 10 11 1 3 1 4 1 5 3 0 2 0 5 4 4 2 0 4 1 3 5 2 0 4 1 3 3 5 5 2 4
120 48 10 12 1 3 1 5 2 3 4 1 0 0 0 5 4 3 4 5 2 1 5 3 4 5 2 1 4 5 4 3 2
121 47 11 1 1 4 0 0 4 3 2 1 5 1 1 1 4 4 3 2 1 5 5 5 4 3 2 0 5 4 3 2 0

```
122 31 11  2 1 4 0 1 5 4 0 2 3 1 2 2 4 5 4 0 2 3 5 0 5 1 3 4 0 5 1 3 4
123 33 11  3 1 4 0 2 3 5 1 0 4 1 3 3 4 3 5 1 0 4 5 4 0 2 1 5 4 0 2 1 5
124 56 11  4 1 4 0 3 1 0 4 5 2 1 4 4 4 1 0 4 5 2 5 2 1 5 0 3 2 1 5 0 3
125 56 11  5 1 4 0 4 2 1 5 3 0 1 5 5 4 2 1 5 3 0 5 3 2 0 4 1 3 2 0 4 1
126 56 11  6 1 4 0 5 0 2 3 4 1 1 0 0 4 0 2 3 4 1 5 1 3 4 5 2 1 3 4 5 2
127 78 11  7 1 4 1 0 4 3 2 1 5 0 1 2 5 5 4 3 2 0 4 5 4 3 2 0 4 5 4 3 5
128 81 11  8 1 4 1 1 5 4 0 2 3 0 2 3 5 0 5 1 3 4 4 0 5 1 3 4 5 4 2 4 5
129 34 11  9 1 4 1 2 3 5 1 0 4 0 3 4 5 4 0 2 1 5 4 4 0 2 1 5 5 5 3 2 4
130 45 11 10 1 4 1 3 1 0 4 5 2 0 4 5 5 2 1 5 0 3 4 2 1 5 0 3 3 2 4 5 4
131 45 11 11 1 4 1 4 2 1 5 3 0 0 5 4 5 3 2 0 4 1 4 3 2 0 4 1 4 3 5 5 2
132 54 11 12 1 4 1 5 0 2 3 4 1 0 0 5 5 1 3 4 5 2 4 1 3 4 5 2 2 4 5 4 3
133 54 12  1 1 5 0 0 5 4 3 2 1 1 1 1 5 5 4 3 2 1 0 0 5 4 3 2 0 5 4 3 2
134 54 12  2 1 5 0 1 3 5 4 0 2 1 2 2 5 3 5 4 0 2 0 4 0 5 1 3 4 0 5 1 3
135 12 12  3 1 5 0 2 4 3 5 1 0 1 3 3 5 4 3 5 1 0 0 5 4 0 2 1 5 4 0 2 1
136 15 12  4 1 5 0 3 2 1 0 4 5 1 4 4 5 2 1 0 4 5 0 3 2 1 5 0 3 2 1 5 0
137 45 12  5 1 5 0 4 0 2 1 5 3 1 5 5 5 0 2 1 5 3 0 1 3 2 0 4 1 3 2 0 4
138 46 12  6 1 5 0 5 1 0 2 3 4 1 0 0 5 1 0 2 3 4 0 2 1 3 4 5 2 1 3 4 5
139 56 12  7 1 5 1 0 5 4 3 2 1 0 1 2 0 0 5 4 3 2 5 0 5 4 3 2 5 4 5 4 3
140 65 12  8 1 5 1 1 3 5 4 0 2 0 2 3 0 4 0 5 1 3 5 4 0 5 1 3 5 5 4 2 4
141 66 12  9 1 5 1 2 4 3 5 1 0 0 3 4 0 5 4 0 2 1 5 5 4 0 2 1 4 5 5 3 2
142 46 12 10 1 5 1 3 2 1 0 4 5 0 4 5 0 3 2 1 5 0 5 3 2 1 5 0 4 3 2 4 5
143 47 12 11 1 5 1 4 0 2 1 5 3 0 5 4 0 1 3 2 0 4 5 1 3 2 0 4 2 4 3 5 5
144 48 12 12 1 5 1 5 1 0 2 3 4 0 0 5 0 2 1 3 4 5 5 2 1 3 4 5 3 2 4 5 4
```

Class	Levels	Values
ROW	12	1 2 3 4 5 6 7 8 9 10 11 12
COL	12	1 2 3 4 5 6 7 8 9 10 11 12
A	2	0 1
B	6	0 1 2 3 4 5
C	2	0 1
D	6	0 1 2 3 4 5

Number of observations 144

Dependent Variable: Y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	143	56628.88889	396.00622	.	.
Error	0	0.00000	.		
Corrected Total	143	56628.88889			

R-Square	Coeff Var	Root MSE	Y Mean
1.000000	.	.	42.77778

Source	DF	Type I SS	Mean Square	F Value	Pr > F
ROW	11	22946.22222	2086.02020	.	.
COL	11	12564.88889	1142.26263	.	.
A*C	1	373.77778	373.77778	.	.
A*D	5	2809.22222	561.84444	.	.
A*C*D	5	676.88889	135.37778	.	.
B*C	5	380.88889	76.17778	.	.
B*D	25	2507.77778	100.31111	.	.
B*C*D	25	5029.44444	201.17778	.	.
A*B*C	5	336.88889	67.37778	.	.
A*B*D	25	4079.44444	163.17778	.	.
A*B*C*D	25	4923.44444	196.93778	.	.

Source	DF	Type III SS	Mean Square	F Value	Pr > F
ROW	0	0.000000	.	.	.
COL	0	0.000000	.	.	.
A*C	1	373.777778	373.777778	.	.
A*D	5	2809.222222	561.844444	.	.
A*C*D	5	676.888889	135.377778	.	.
B*C	5	380.888889	76.177778	.	.
B*D	25	2507.777778	100.311111	.	.
B*C*D	25	5029.444444	201.177778	.	.
A*B*C	5	336.888889	67.377778	.	.
A*B*D	25	4079.444444	163.177778	.	.
A*B*C*D	25	4923.444444	196.937778	.	.

Class	Levels	Values
ROW	12	1 2 3 4 5 6 7 8 9 10 11 12
COL	12	1 2 3 4 5 6 7 8 9 10 11 12
A	2	0 1
B	6	0 1 2 3 4 5
C	2	0 1
D	6	0 1 2 3 4 5
F1	2	0 1
F2	6	0 1 2 3 4 5
F3	6	0 1 2 3 4 5
F4	6	0 1 2 3 4 5
F5	6	0 1 2 3 4 5
F6	6	0 1 2 3 4 5
F7	6	0 1 2 3 4 5
F8	6	0 1 2 3 4 5
F9	6	0 1 2 3 4 5
F10	6	0 1 2 3 4 5
F11	6	0 1 2 3 4 5
F12	6	0 1 2 3 4 5

F13	6	0 1 2 3 4 5
F14	6	0 1 2 3 4 5
F15	6	0 1 2 3 4 5
F16	6	0 1 2 3 4 5
F17	6	0 1 2 3 4 5
F18	6	0 1 2 3 4 5
F19	6	0 1 2 3 4 5
F20	6	0 1 2 3 4 5
F21	6	0 1 2 3 4 5
F22	6	0 1 2 3 4 5
F23	6	0 1 2 3 4 5
F24	6	0 1 2 3 4 5
F25	6	0 1 2 3 4 5

Number of observations 144

Dependent Variable: Y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	143	56628.88889	396.00622	.	.
Error	0	0.00000	.		
Corrected Total	143	56628.88889			

R-Square	Coeff Var	Root MSE	Y Mean
1.000000	.	.	42.77778

Source	DF	Type I SS	Mean Square	F Value	Pr > F
ROW	11	22946.22222	2086.02020	.	.
COL	11	12564.88889	1142.26263	.	.
F1	1	373.77778	373.77778	.	.
F2	5	2809.22222	561.84444	.	.
F3	5	676.88889	135.37778	.	.
F4	5	380.88889	76.17778	.	.
F5	5	349.88889	69.97778	.	.
F6	5	820.75568	164.15114	.	.
F7	5	439.89186	87.97837	.	.
F8	5	466.79189	93.35838	.	.
F9	5	430.44946	86.08989	.	.
F10	5	1445.72222	289.14444	.	.
F11	5	1432.76419	286.55284	.	.
F12	5	864.17976	172.83595	.	.
F13	5	499.91507	99.98301	.	.
F14	5	786.86321	157.37264	.	.
F15	5	336.88889	67.37778	.	.
F16	5	360.22222	72.04444	.	.

F17	5	1667.19654	333.43931	.	.
F18	5	681.77136	136.35427	.	.
F19	5	65.63554	13.12711	.	.
F20	5	1304.61879	260.92376	.	.
F21	5	358.05556	71.61111	.	.
F22	5	669.29465	133.85893	.	.
F23	5	1315.06945	263.01389	.	.
F24	5	1271.66476	254.33295	.	.
F25	5	1309.36002	261.87200	.	.

Dependent Variable: Y

Source	DF	Type III SS	Mean Square	F Value	Pr > F
ROW	11	15980.56347	1452.77850	.	.
COL	11	10673.99754	970.36341	.	.
F1	1	3.23892	3.23892	.	.
F2	5	1540.97933	308.19587	.	.
F3	5	676.88889	135.37778	.	.
F4	5	318.92506	63.78501	.	.
F5	5	1944.81880	388.96376	.	.
F6	5	506.04048	101.20810	.	.
F7	5	1268.86555	253.77311	.	.
F8	5	1247.25087	249.45017	.	.
F9	5	678.34149	135.66830	.	.
F10	5	1444.89182	288.97836	.	.
F11	5	628.53206	125.70641	.	.
F12	5	895.05266	179.01053	.	.
F13	5	931.13111	186.22622	.	.
F14	5	639.77744	127.95549	.	.
F15	5	336.88889	67.37778	.	.
F16	5	1427.39890	285.47978	.	.
F17	5	755.48818	151.09764	.	.
F18	5	991.66336	198.33267	.	.
F19	5	1693.33709	338.66742	.	.
F20	5	1285.33895	257.06779	.	.
F21	5	1180.59767	236.11953	.	.
F22	5	985.22324	197.04465	.	.
F23	5	1399.47297	279.89459	.	.
F24	5	1795.88530	359.17706	.	.
F25	5	1309.36002	261.87200		

5. A Class of SSOLS(q, q - 1)

In the previous section, a form of a Kronecker product of a 2×2 and of a 3×3 Latin square was used to form a Latin square of side six. The set of five 6×6 Latin squares obtained forms a sum of squares orthogonal set that is denoted as SSOLS(6, 5) set. This form of the Latin square is not unique for obtaining a SSOLS(6, 5) set. For example, the following form of a 6 × 6 Latin square also results in a SSOLS(6, 5) set:

L1	L2	L3	L4	L5
0 1 2 3 4 5	0 1 2 3 4 5	0 1 2 3 4 5	0 1 2 3 4 5	0 1 2 3 4 5
1 2 3 4 5 0	5 0 1 2 3 4	4 5 0 1 2 3	3 4 5 0 1 2	2 3 4 5 0 1
2 3 4 5 0 1	1 2 3 4 5 0	5 0 1 2 3 4	4 5 0 1 2 3	3 4 5 0 1 2
3 4 5 0 1 2	2 3 4 5 0 1	1 2 3 4 5 0	5 0 1 2 3 4	4 5 0 1 2 3
4 5 0 1 2 3	3 4 5 0 1 2	2 3 4 5 0 1	1 2 3 4 5 9	5 0 1 2 3 4
5 0 1 2 3 4	4 5 0 1 2 3	3 4 5 0 1 2	2 3 4 5 0 1	1 2 3 4 5 0

The squares are formed by constructing a cyclical Latin square for the first square. Then the last five rows of the first square are cyclically permuted to obtain square two. This process is continued to obtain the other three squares. Using 36 observations for the combinations of rows (B) and columns (D), the degrees of freedom and sums of squares for the set in the previous section are presented in Table 5.1 and those for the above example in Table 5.2. The results in these tables show that these sets both form SSOLS(6, 5) sets as all of the degrees of freedom and Type I sums of squares are taken into account.

Table 5.1. Analysis of variance for the SSOLS(6, 5) set in Section 4.

Source of variation	Degrees of freedom	Sum of squares	Latin square	Degrees of freedom	Sum of squares
Total	36	1605			
Mean	1	1381.3611			
B	5	12.1389			
D	5	36.0856			Type I
B × D	25	174.6944	L1	5	24.4722
			L2	5	49.1231
			L3	5	10.8433
			L4	5	64.9454
			L5	5	25.3104
					Type III
			L1	5	7.6639
			L2	5	13.1994
			L3	5	47.4117
			L4	5	64.6597
			L5	5	25.3104

Table 5.2. Analysis of variance for the SSOLS(6, 5) set in this section .

Source of variation	Degrees of freedom	Sum of squares	Latin square	Degrees of freedom	Sum of squares
Total	36	1605			
Mean	1	1381.3611			

B	5	12.1389			
D	5	36.8056			Type I
B × D	25	174.6944	L1	5	45.1389
			L2	5	22.1700
			L3	5	20.7667
			L4	5	64.4096
			L5	5	22.2092
					Type III
			L1	5	61.1549
			L2	5	17.3569
			L3	5	56.9668
			L4	5	46.3910
			L5	5	22.2092

In both cases, the sum of the degrees of freedom and of the Type I sum of the sums of squares for the five Latin squares add to that for the B × D interaction. Thus, they are SSOLSs.

If we use the following 10 × 10 Latin square

0	1	2	3	4	5	6	7	8	9
1	2	3	4	0	6	7	8	9	5
2	3	4	0	1	7	8	9	5	6
3	4	0	1	2	8	9	5	6	7
4	0	1	2	3	9	5	6	7	8
5	6	7	8	9	0	1	2	3	4
8	9	5	6	7	2	3	4	0	1
6	7	8	9	5	4	0	1	2	3
9	5	6	7	8	1	2	3	4	0
7	8	9	5	6	3	4	0	1	2

and cyclically permute the last nine rows, a SSOLS(10, 9) set is formed. The sums of squares for the symbols in the ten Latin squares and the degrees of freedom add to that for the row by column interaction sum of squares with 81 degrees of freedom. The results for a 10 × 10 Latin square with 100 observations are presented in Table 5.3.

Table 5.3. Analysis of variance for the SSOLS(10, 9) set.

Source of variation	Degrees of freedom	Sum of squares	Latin square	Degrees of freedom	Sum of squares
Total	100	18934			
Mean	1	18333.16			
B (row)	9	141.44			
D (column)	9	18.24			Type I
B × D	81	441.16	L1	9	58.0124
			L2	9	67.8808
			L3	9	66.8265
			L4	9	29.7700

L5	9	16.9558
L6	9	19.5151
L7	9	121.8063
L8	9	12.3135
L9	9	48.0780

The sum of the degrees of freedom and of the Type I sums of squares add to that for the B $\times$ D interaction. Hence, this set forms a SSOLS(10, 9) set.

In order to obtain complete sets of sum of squares orthogonal F-squares, the SSOLS(q, q - 1) or MOLS(pk, pk - 1), p a prime number, sets are required. The MOLS(4, 3) set was used for n = 2 $\times$ 4 and n = 3 $\times$ 4 cases and the SSOLS(6, 5) set was used for n = 2 $\times$ 6. It is conjectured that the following theorem is true:

Theorem: *Complete sets of sum of squares orthogonal F-squares are available for all sets of SSOLS(q, q - 1) and MOLS(p^k, p^k - 1).*

It appears that the above method of constructing SSOLS(q, q - 1) sets will work for any value of q. When n = p^k, p a prime number, a MOLS(p^k, p^k - 1) set is available. MOLS sets are combinatorially orthogonal as well as being SSOLS sets. SSOLS sets represent an advance in presently available theory. With the availability of SSOLS sets, the number of complete sets of sum of squares orthogonal F-squares is greatly expanded for use in such areas as constructing codes, fractional replication, etc.. The results given herein represent a sizeable addition to the new geometry proposed by Federer (2003).

6. LITERATURE CITED

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